Transport signatures of strange metals

ASCES Summer School

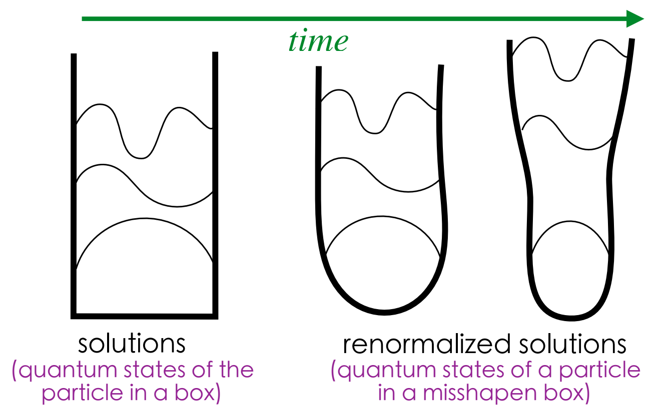
Contents

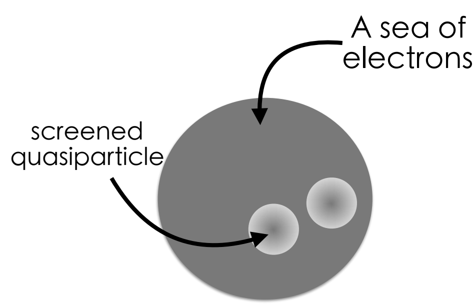
- Crash course in strange metals
- Some classical examples of continuous phase transitions
- Extension to quantum critical phase transitions.
- Connections to the strange metal
- The elephant in the room

A crash course in Strange metals

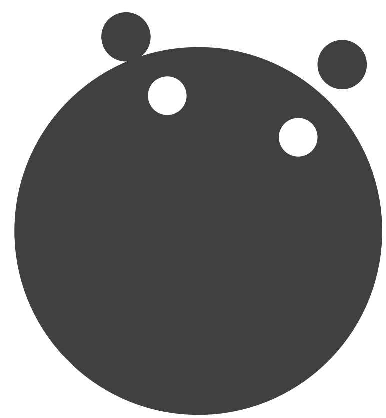
Normal metals: the long-lived quasiparticle

Adiabatic evolution





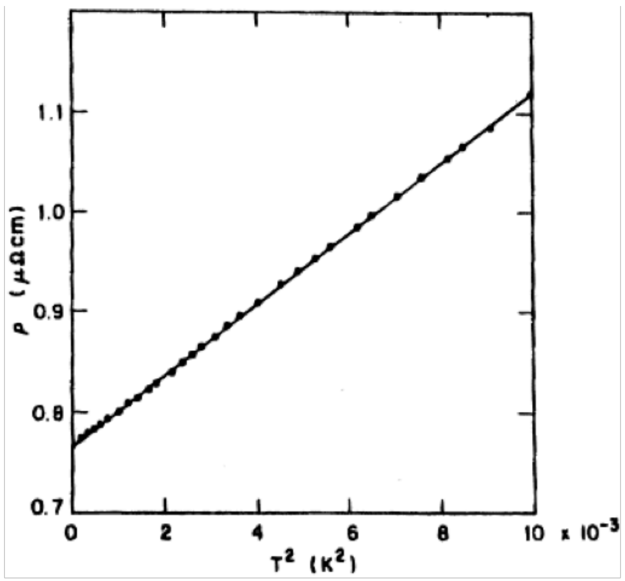
Screening leading to effectively weaker interactions between quasiparticles



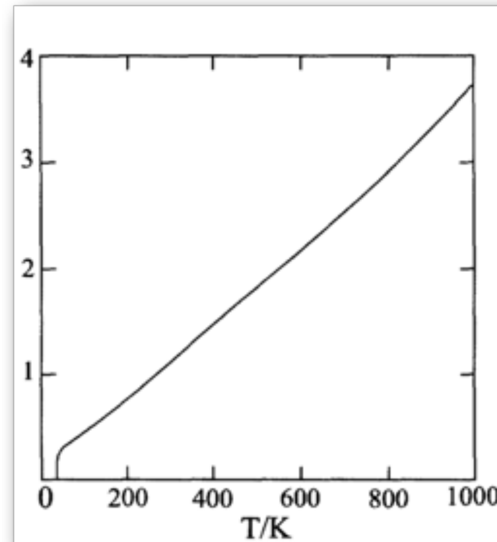
$$\frac{\hbar}{\tau} \propto (k_B T)^2$$

Fermi liquid theory

T-linear resistivity & the breakdown of the quasiparticle picture



K. Andres, J.E. Graebner, H.R. Ott, Phys. Rev. Lett. 1975

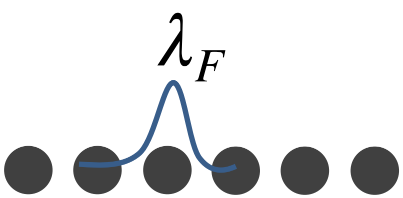


Takagi et al (1994)

$$\frac{\hbar}{\tau} \propto (k_B T)^2$$

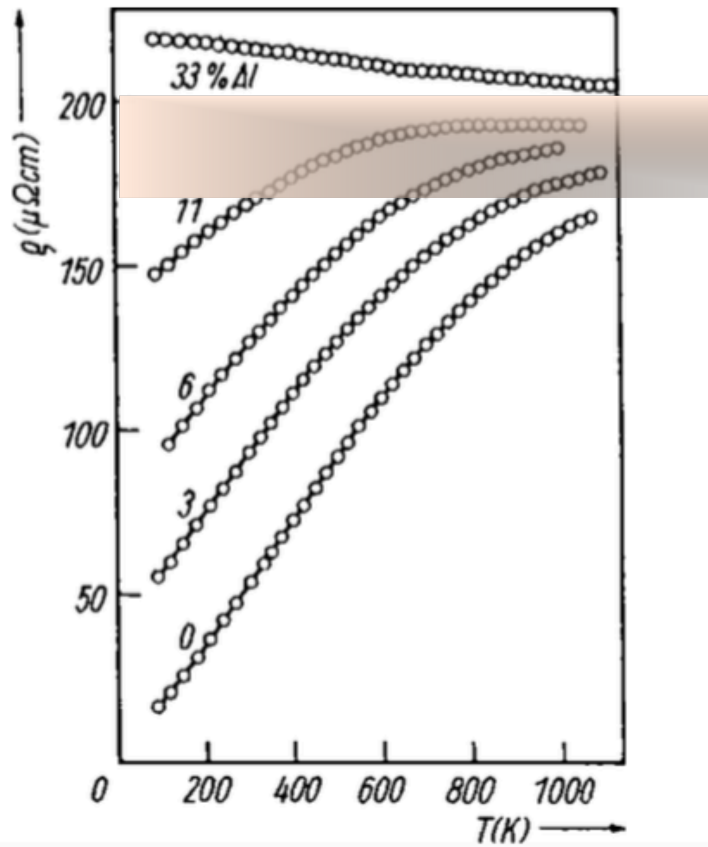
$$\frac{\hbar}{\tau} = k_B T$$

T-linear resistivity & the breakdown of the quasiparticle picture

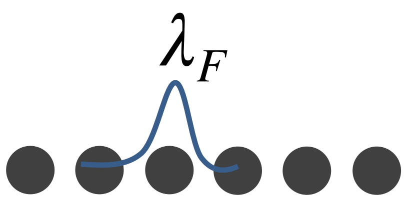


$k_F l_{min} \sim 1$

Mott-Ioffe-Regel limit

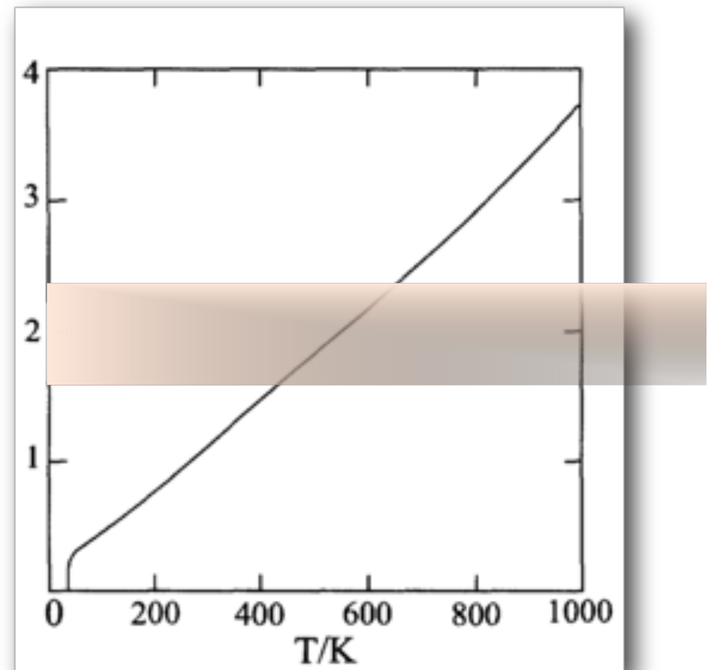


T-linear resistivity & the breakdown of the quasiparticle picture

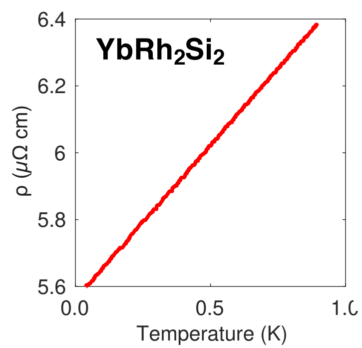


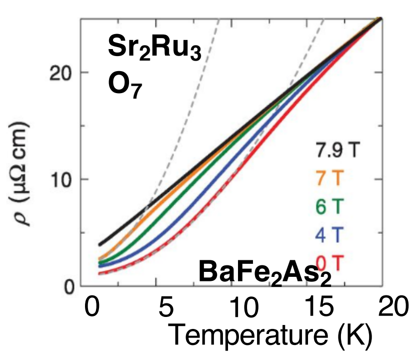
$k_F l_{min} \sim 1$

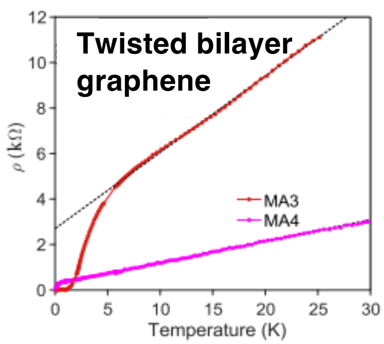
Mott-Ioffe-Regel limit

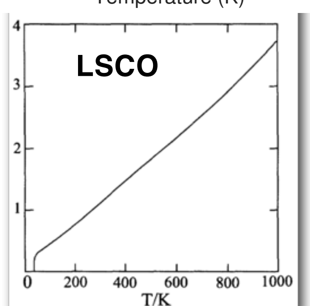


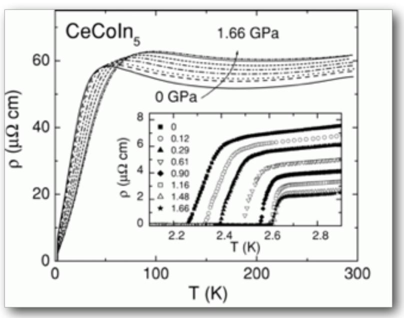
T-linear resistivity & the absence of quasiparticles

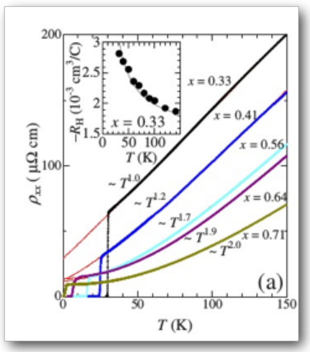












Critical scaling near a phase transition

Classical continuous phase transitions

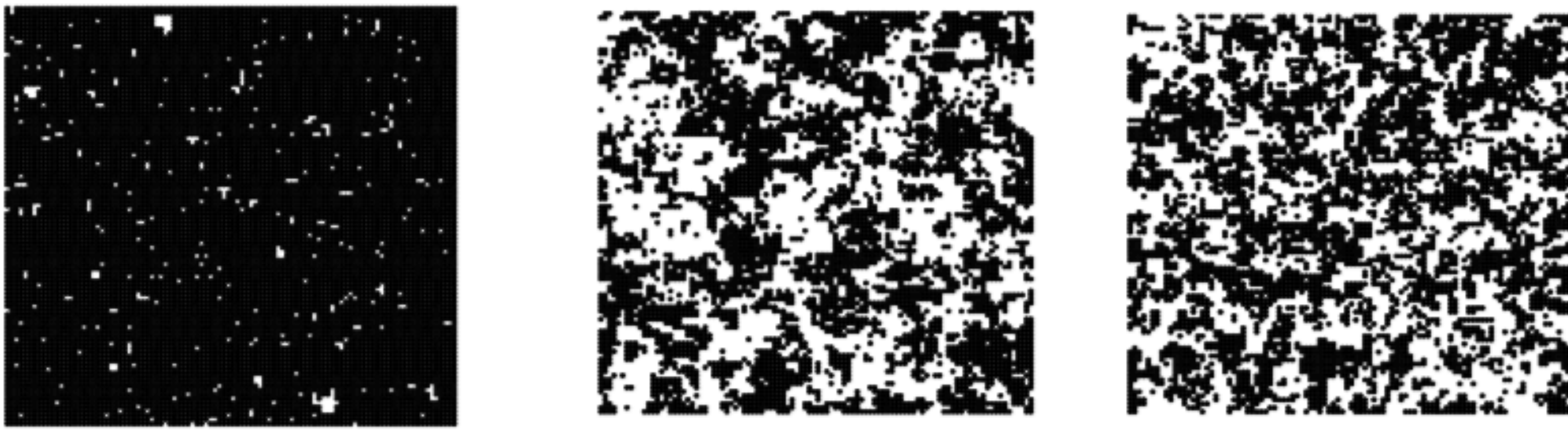
CONTINUOUS PHASE TRANSITIONS

Many symmetry-breaking transitions, ferromagnets, metal-superconductor transitions, nematic transitions etc..

DISCONTINUOUS ORDER PHASE TRANSITIONS

L
p
ferroelectrics etc..

Classical continuous phase transition

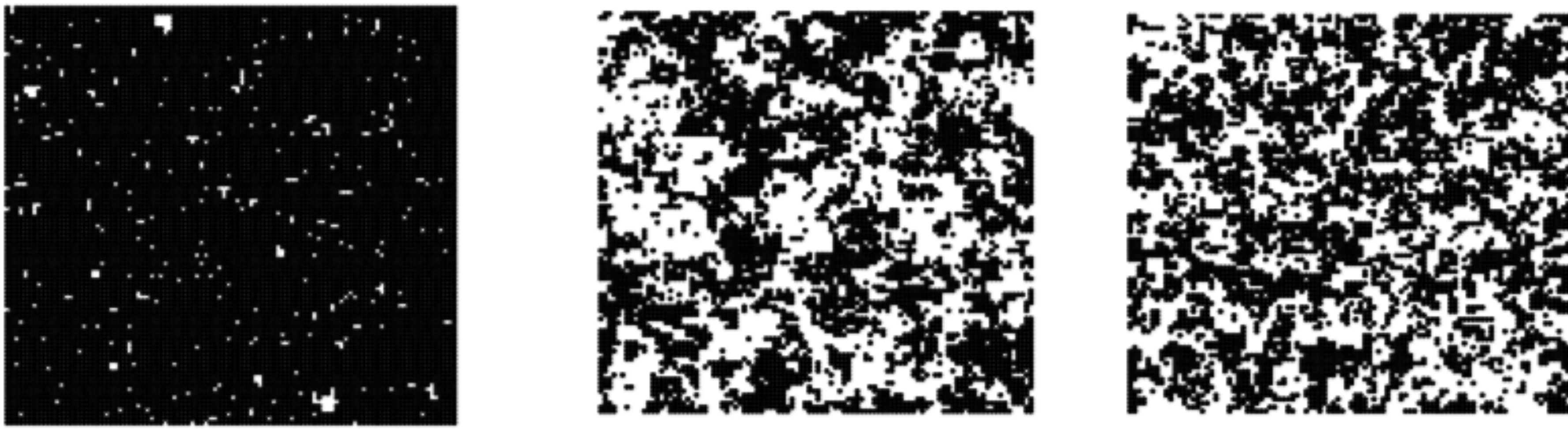


$T < T_c$ $\quad$ $T \approx T_c$ $\quad$ $T > T_c$

- Phase transitions whose order parameter vanishes continuously at T_c

Classical continuous phase transition

e.g. Density-density transition ■ Density 1 □ Density 2

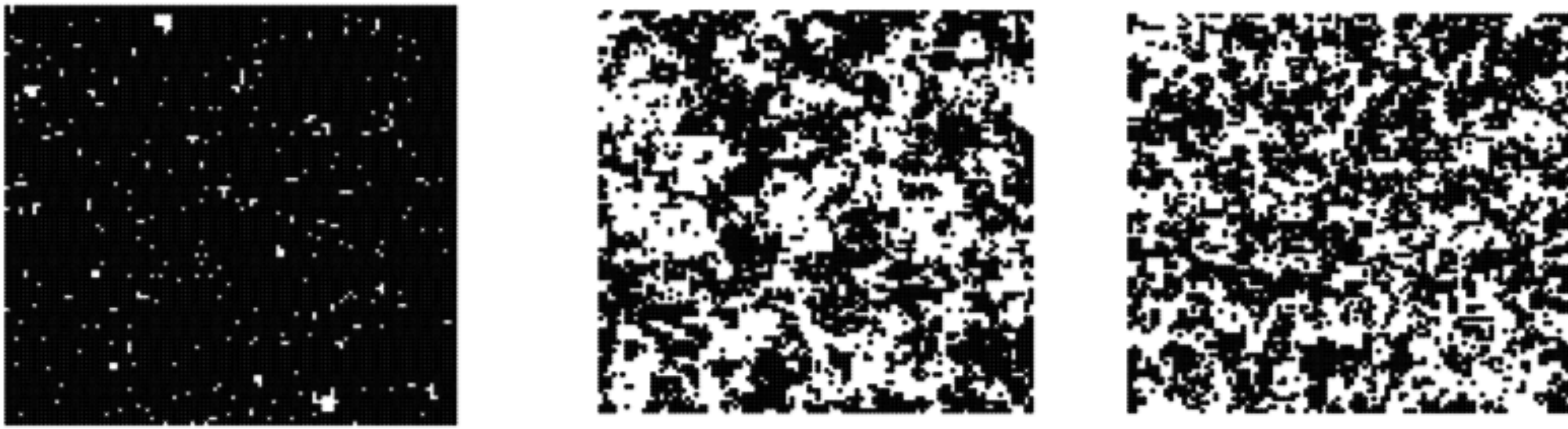


$T < T_c$ $T \approx T_c$ $T > T_c$

- Above T_c there are fluctuations of the order parameter, that lead to correlated regions with an average length scale ξ. This scale diverges at T_c.

Classical continuous phase transition

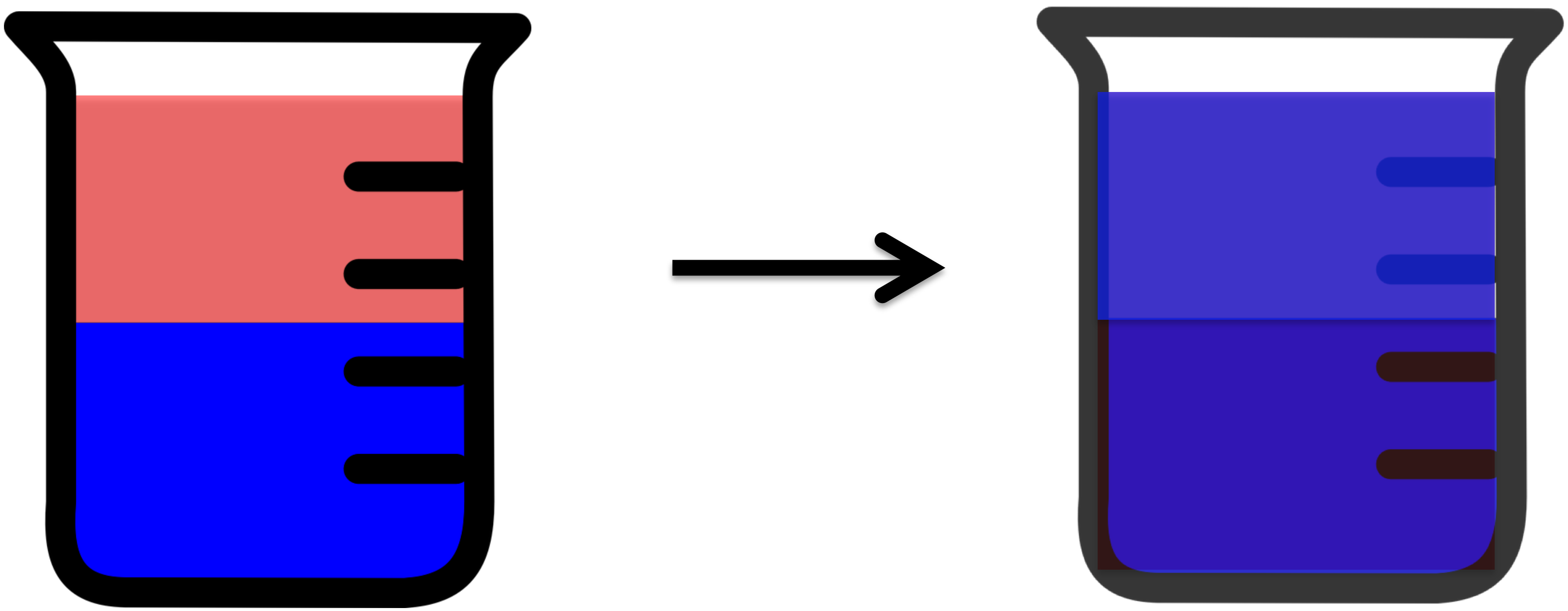
e.g. Paramagnet-ferromagnet transition Spin up Spin down



$T < T_c$ $T \approx T_c$ $T > T_c$

- Above T_c there are fluctuations of the order parameter, that lead to correlated regions with an average length scale ξ. This scale diverges at T_c.

Critical opalescence



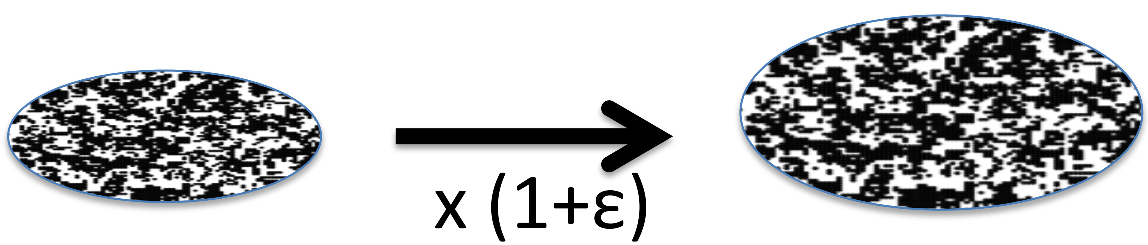
$T<T_c$ $T>T_c$

Heptane + ethanol

Critical Opalescence

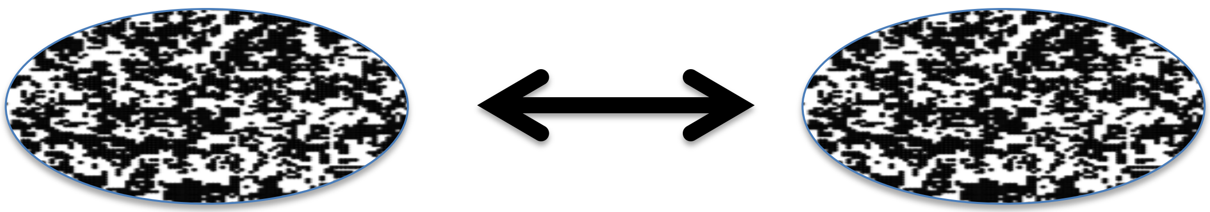
Fluctuations and self-similarity

- These fluctuations lead to a striking property of self-similarity. This is an emergent (statistical) symmetry, that the system is invariant under a change of length scale.



Fluctuations and self-similarity

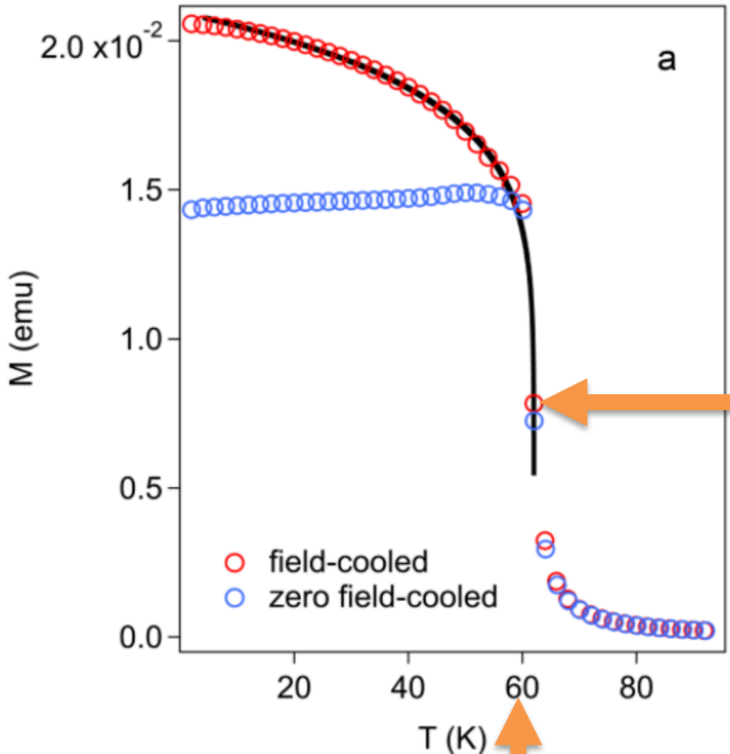
- One neat result is that the system at some temperature near T_c, will look similar to the system at some temperature slightly further away from T_c.



x $(1+\varepsilon)$ $(T=T_c+\delta)$ x $(1+\varepsilon/\nu)$ $(T=T_c+\delta')$

- And since such fluctuations affect essentially all response functions (heat capacity, magnetic susceptibility etc.), this self similarity is inherited by these properties in the critical exponents.

Scaling phenomena in ferromagnets



$M(T_c)$

T_c

Xiaodong Xu, Nature 2017

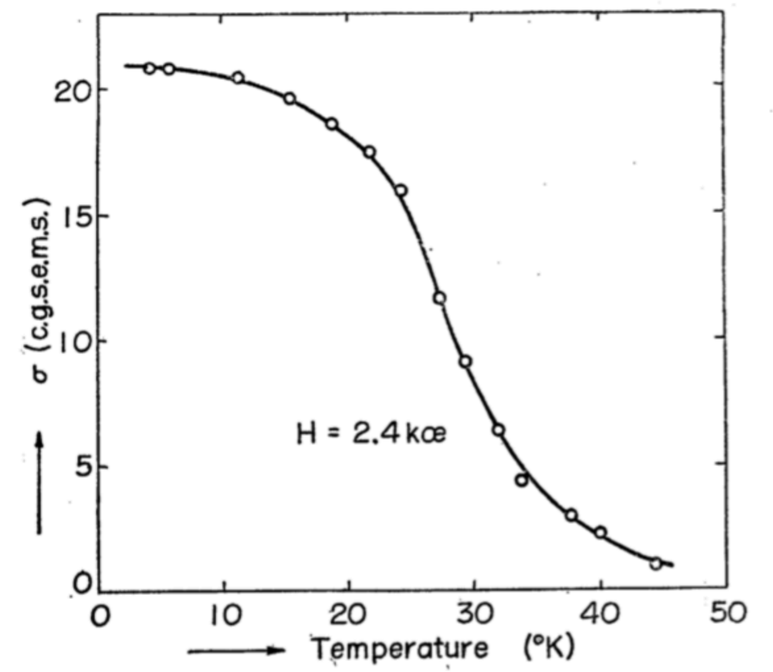
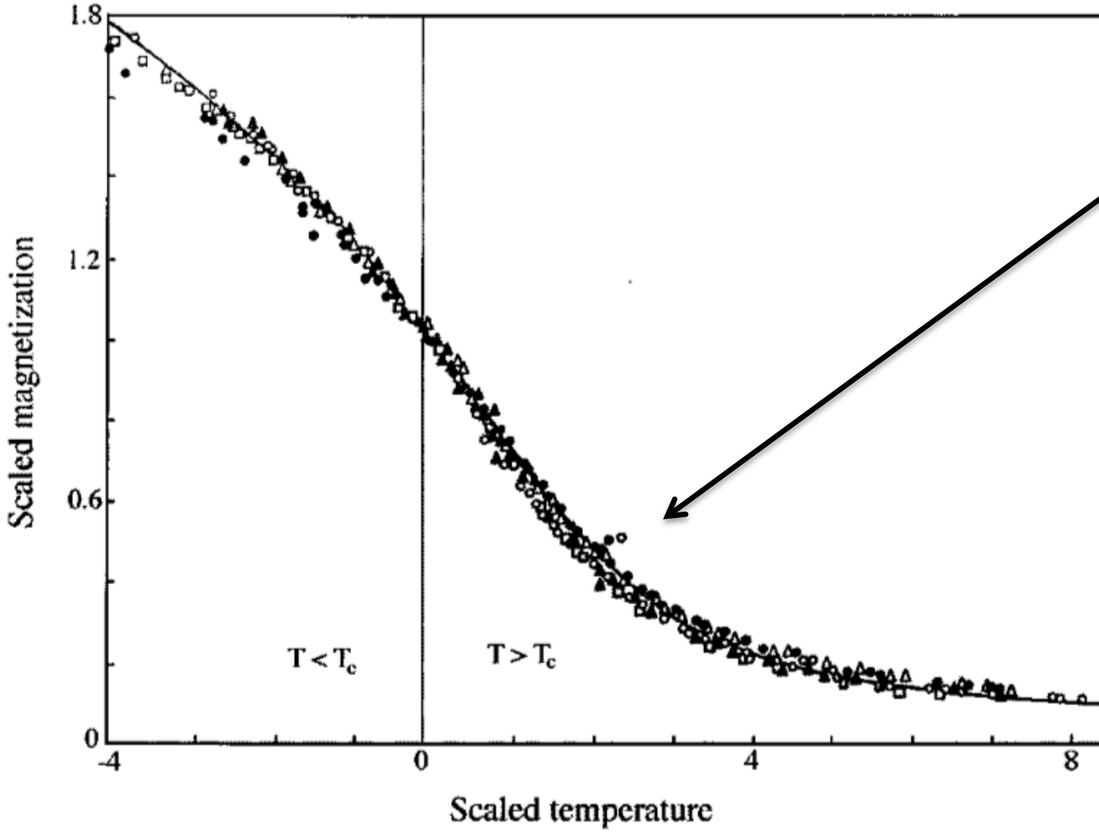


Fig. 2. Temperature dependence of magnetization.

Tsubokawa, JPSJ 1960

Scaling phenomena in ferromagnets

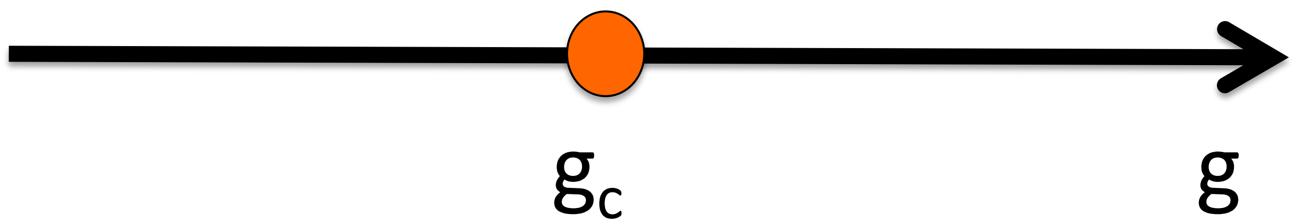


Data is from 5 different FMs, all of which collapse onto a single curve because they share the same critical exponents.

- One example is that the exponents of how these responses vary in temperature have power laws that group continuous phase transitions in universality classes. One neat consequence is the ability to collapse data from different systems, or as a function of different variables, onto a single curve.

Continuous quantum phase transitions

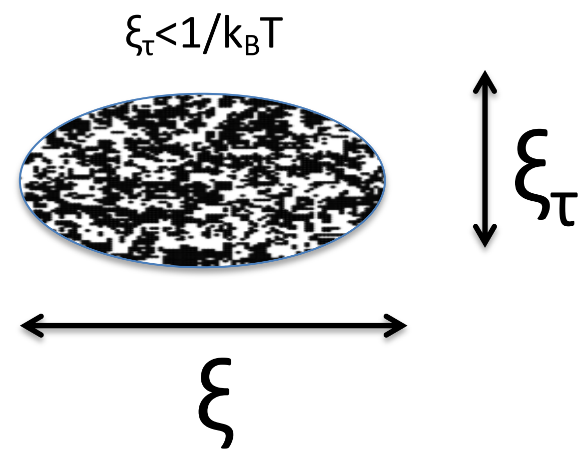
Quantum critical phase transitions

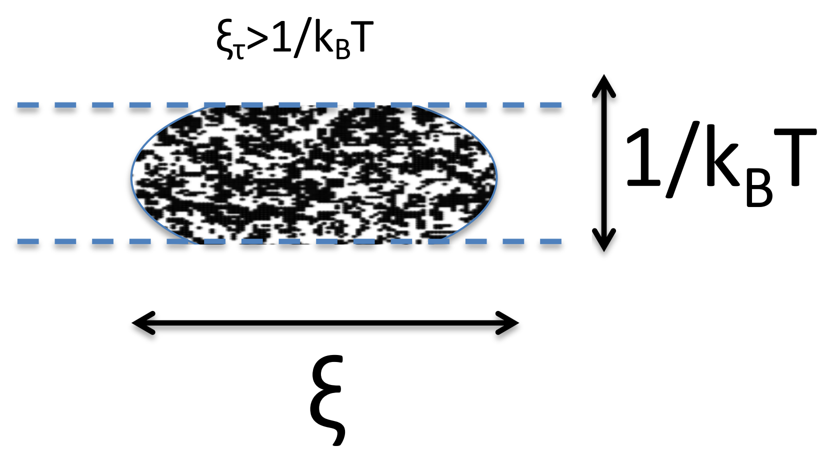


- T=0 phase transitions as a function of non-thermal parameter g_c
- g can be magnetic field, pressure, carrier density, strain, electric field……

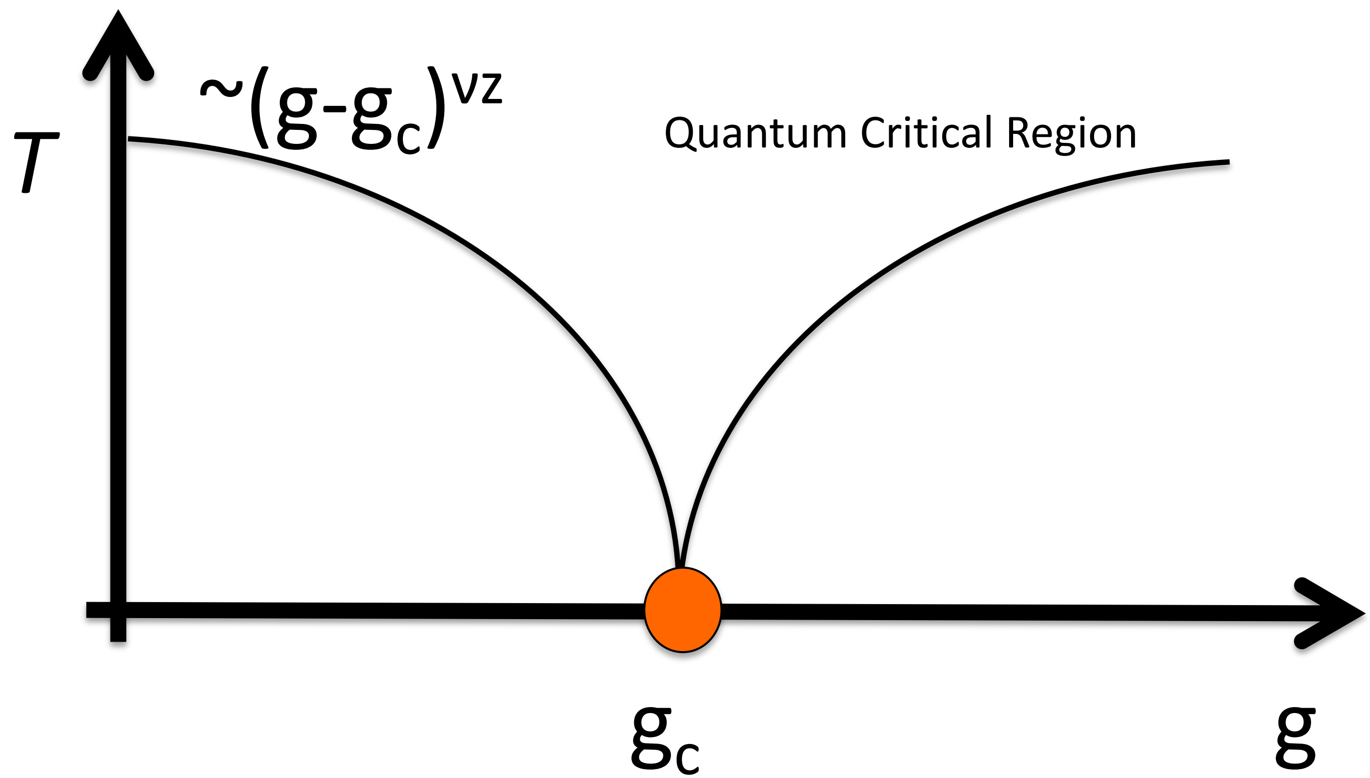
Quantum critical phase transitions

- Quantum fluctuations in time have correlation length ξ_τ, and this scale diverges at the transition just like ξ.
- Time can be treated like another dimension, so that a d dimensional QPT is equivalent to a d+1 dimensions.
- So we expect to see some of the same scaling behavior as we saw in classical transitions.
- Temperature plays a special role, providing an intrinsic cutoff to the time-dimension, and therefore to the coherence of quantum fluctuations.
- We expect interesting scaling behavior at quantum critical points too.





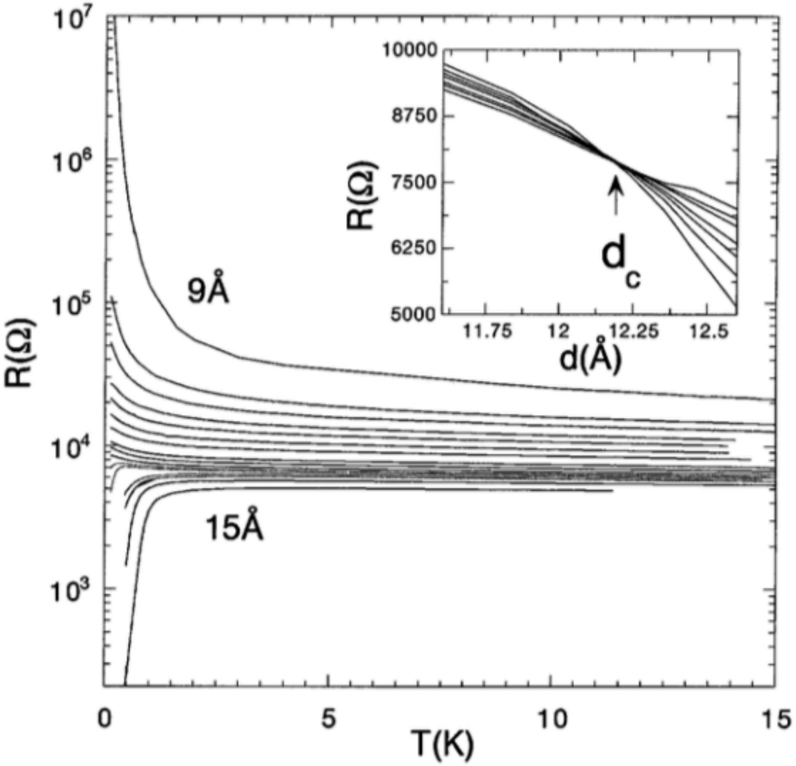
A generic T-g phase diagram for a quantum critical system

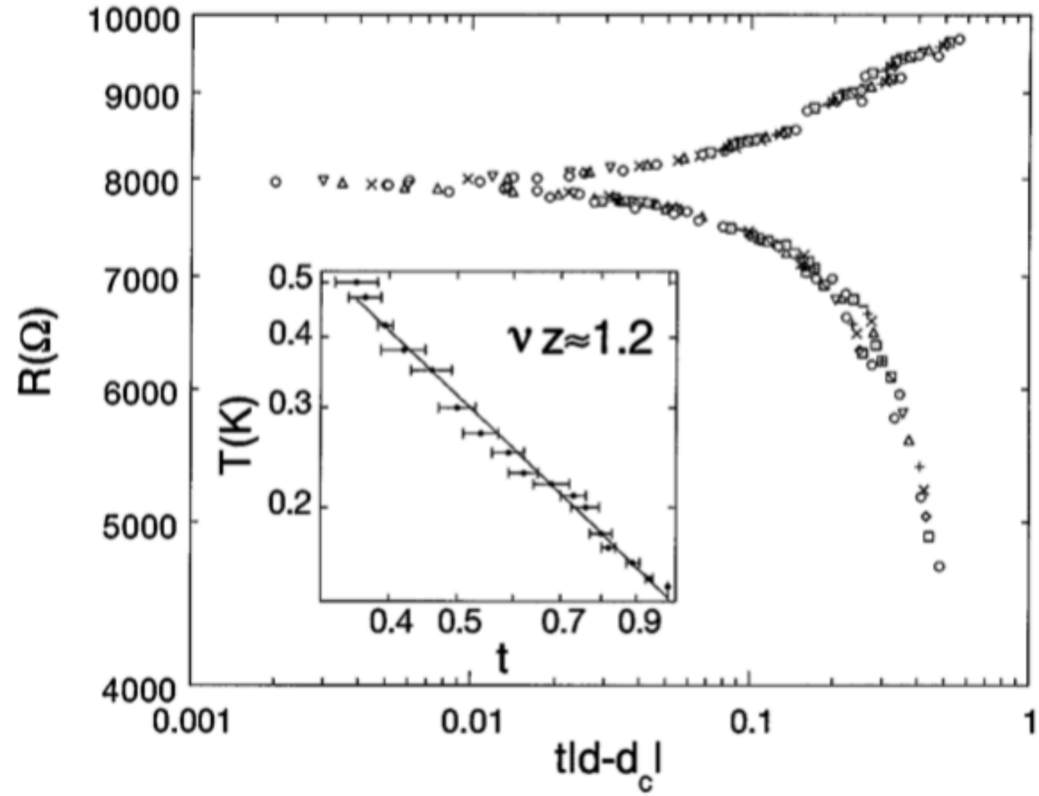


With νz being the connected to critical exponents of the QPT. Note that at g_c, temperature is the only scale in the system, so the quantum fluctuations dominate at all temperatures.

A simple example

Superconductor-Insulator transition in thin film amorphous Bi





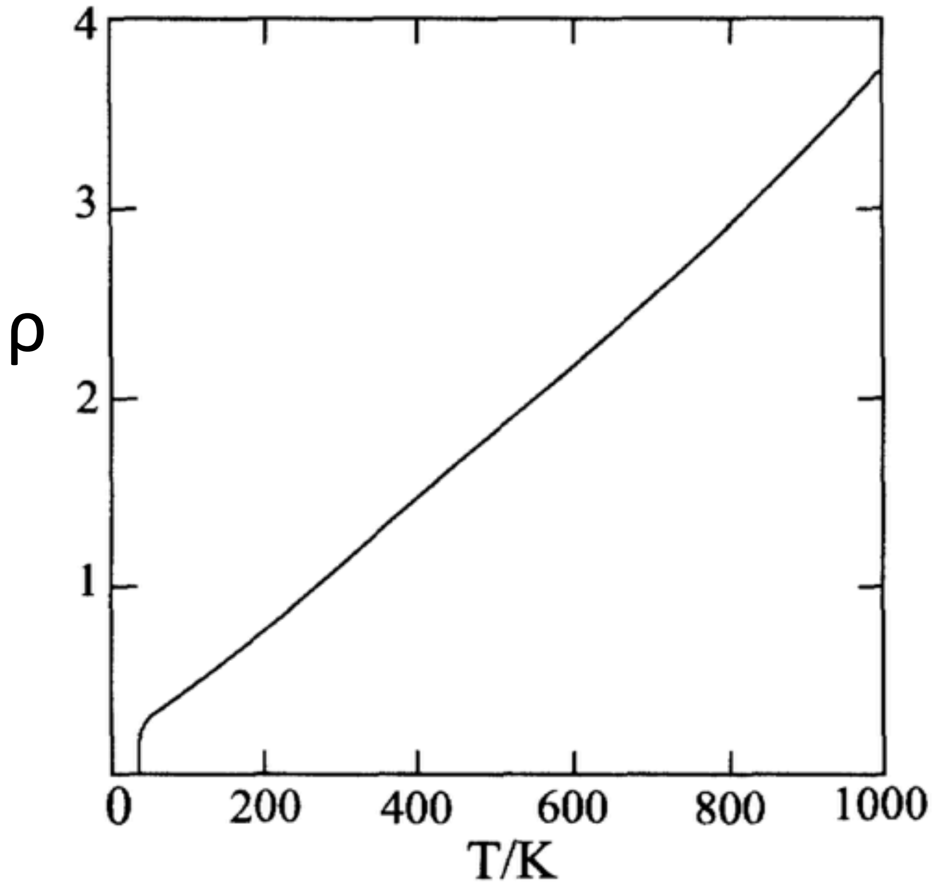
N. Marković, C. Christiansen, and A. M. Goldman PRL 1998

In this case, the non-thermal parameter g is thickness d. At some critical thickness d_c the system goes from being an insulator to a superconductor.

Connections to strange metals

Are the charge carriers metallic quasiparticles?

$La_{1-x}Sr_xCuO_4$



Takagi PRL 1992

Are the charge carriers metallic quasiparticles?

In the theory of the Fermi liquid, the average excitation has energy $k_B T$. It must be long-lived so we require that….

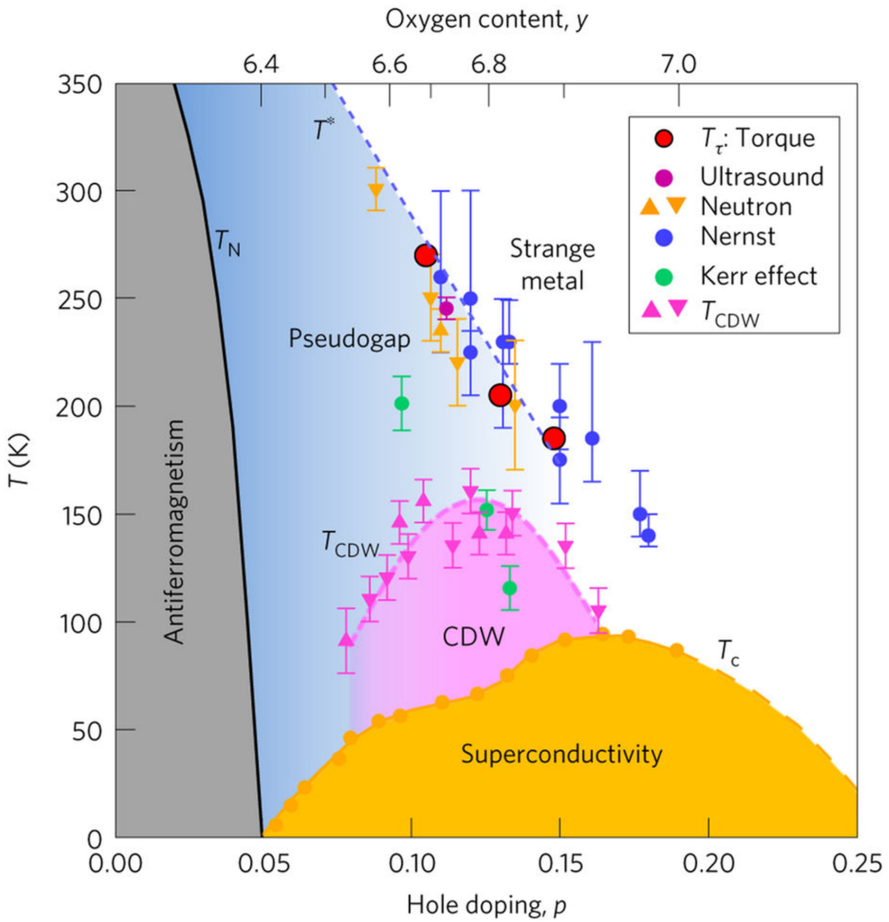
$$\frac{\hbar}{\tau} \ll k_B T$$

…. and since the average scattering energy goes as…..

$$\frac{\hbar}{\tau} \propto (k_B T)^2$$

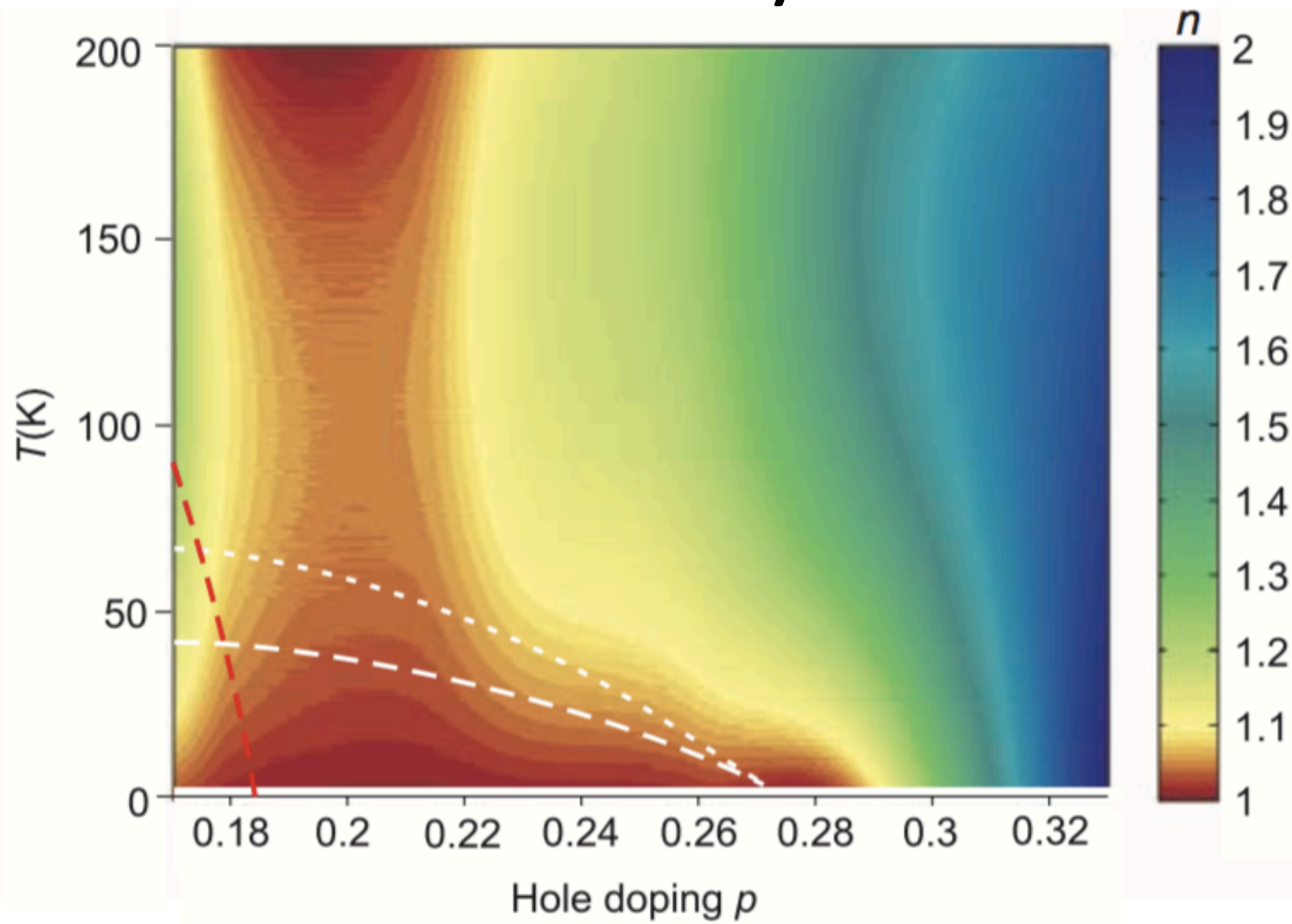
- Thus at low enough temperatures such quasiparticles can always be found, and roughly speaking the resistivity should go as T^2.
- So it is quite a surprise that T-linear resistivity persists to the lowest temperatures measured in these systems.

The cuprate phase diagram is tuned by doping *p* holes, g=*p*



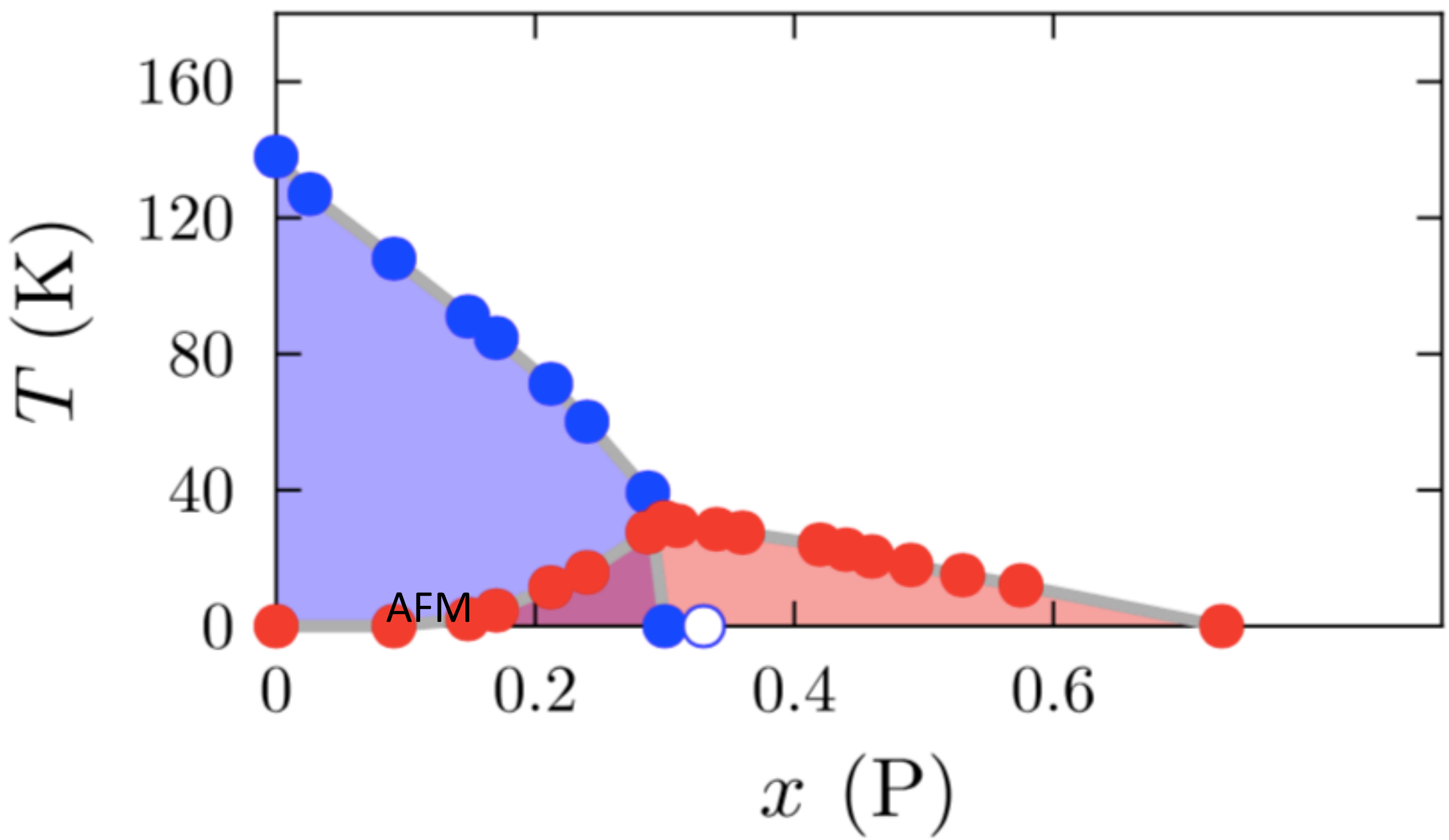
A large number of measurements show evidence for a pseudogap phase boundary that terminates somewhere in the superconducting dome, often near optimal doping.

The phase diagram – power law of the resistivity



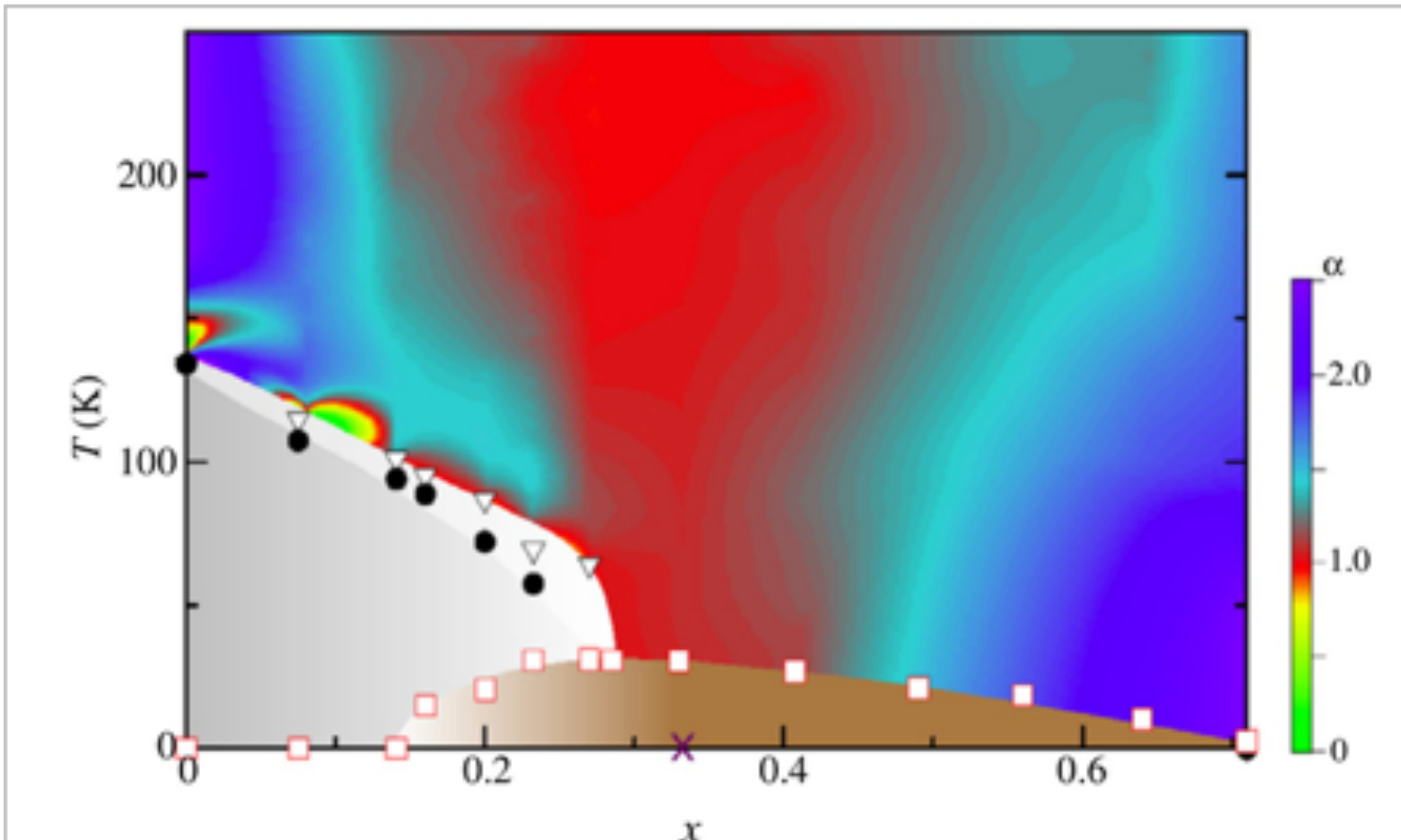
The resistance also appears strongly linear near these critical dopings, crossing over to more Fermi liquid behavior at high *p.*

Other examples: Iron-pnictides e.g. $BaFe_2(As_{1-x}P_x)_2$



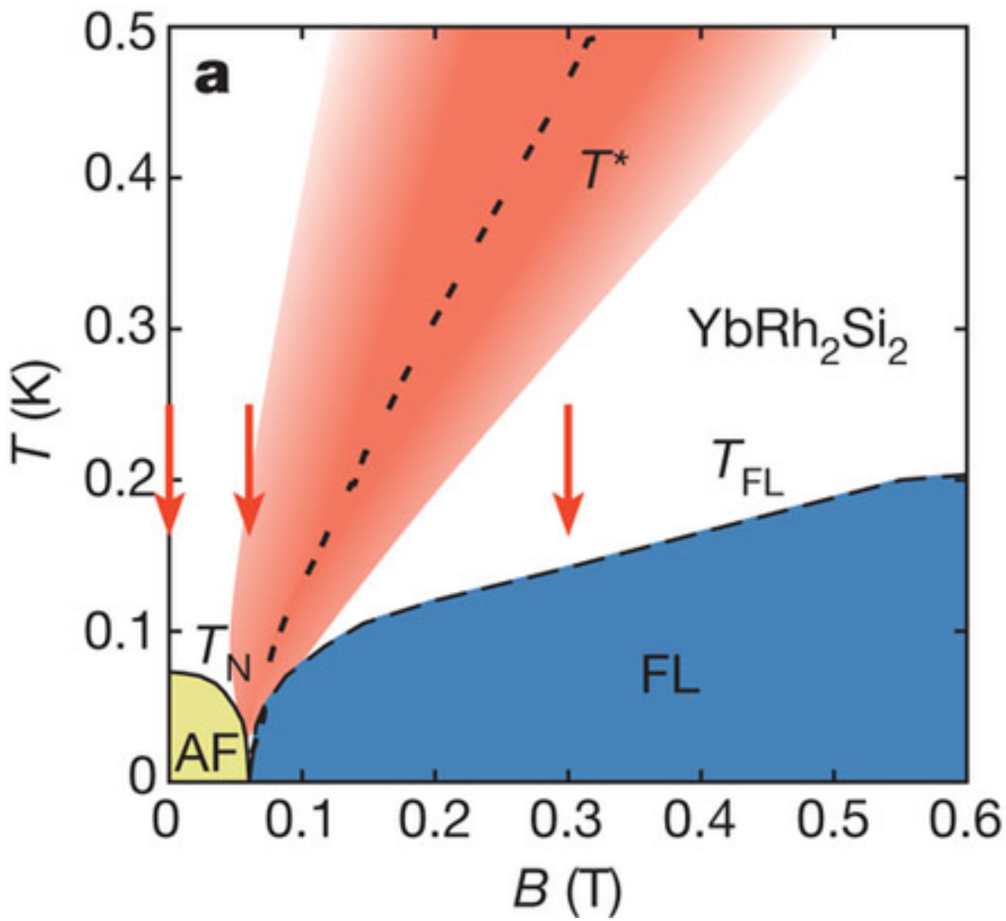
But what is g? The parameter being tuned in P replacing As, but it is unclear what affect this have. It is changing both carrier density and unit cell size, like a kind of chemical pressure.

Other examples: Iron-pnictides e.g. $BaFe_2(As_{1-x}P_x)_2$



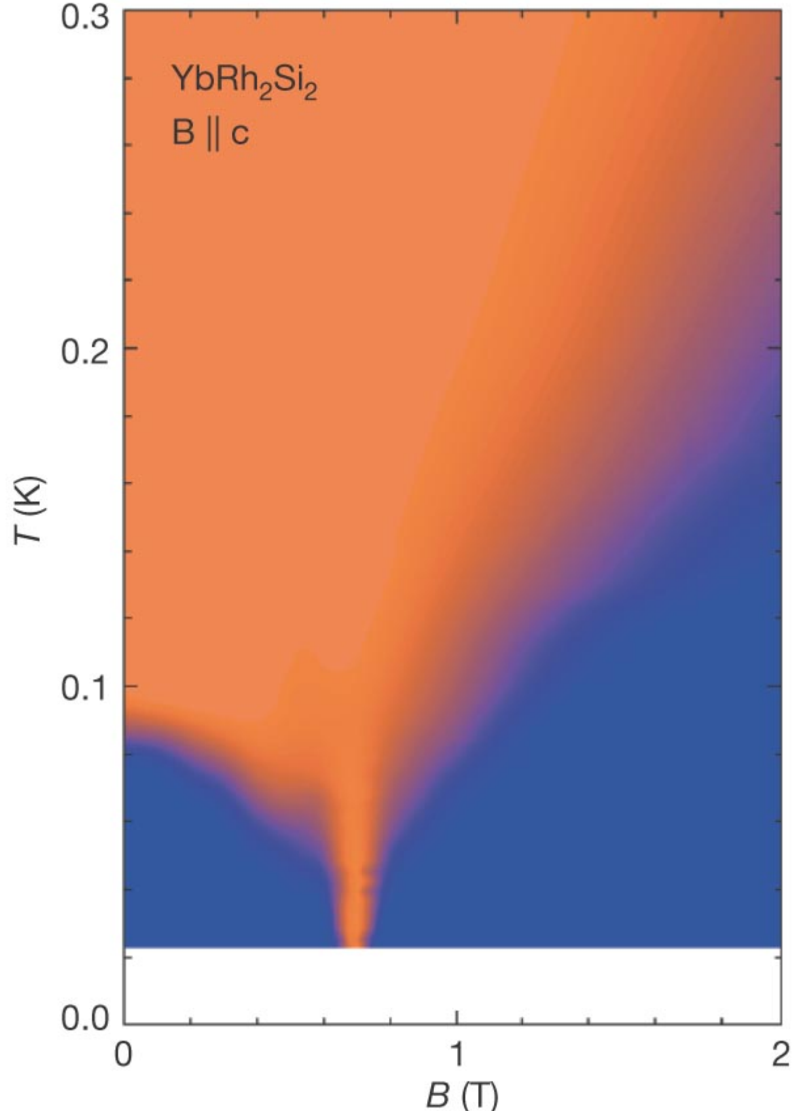
What is observed is a similar kind of phenomenology, one where the resistance is most linear near the termination of the AFM phase boundary.

Other examples: Heavy fermion $YbRh_2Si_2$

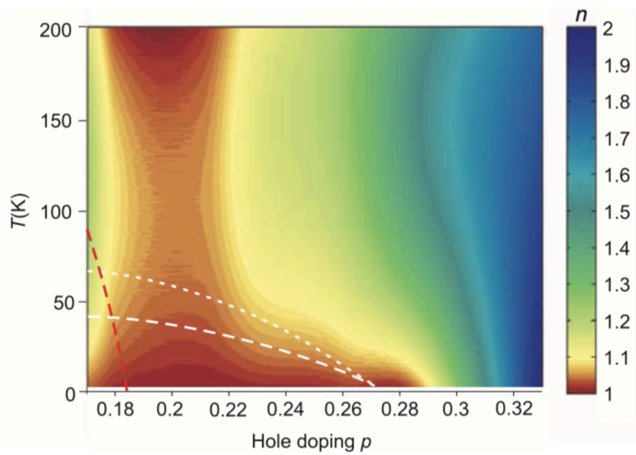


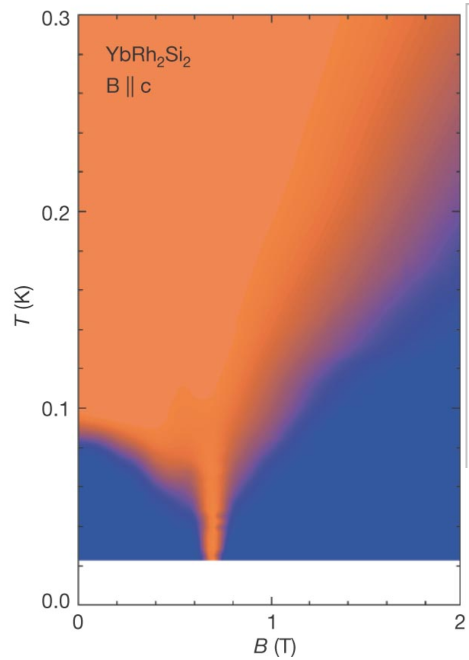
Here the parameter being tuned is magnetic field (in the c-axis), so that g=B.

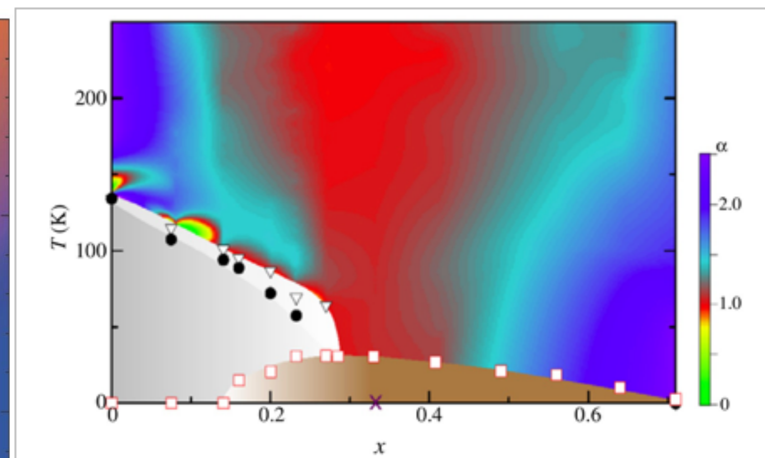
Other examples: Heavy fermion $YbRh_2Si_2$

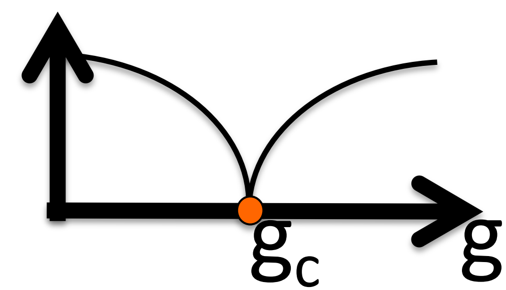


We notice something intriguing these examples







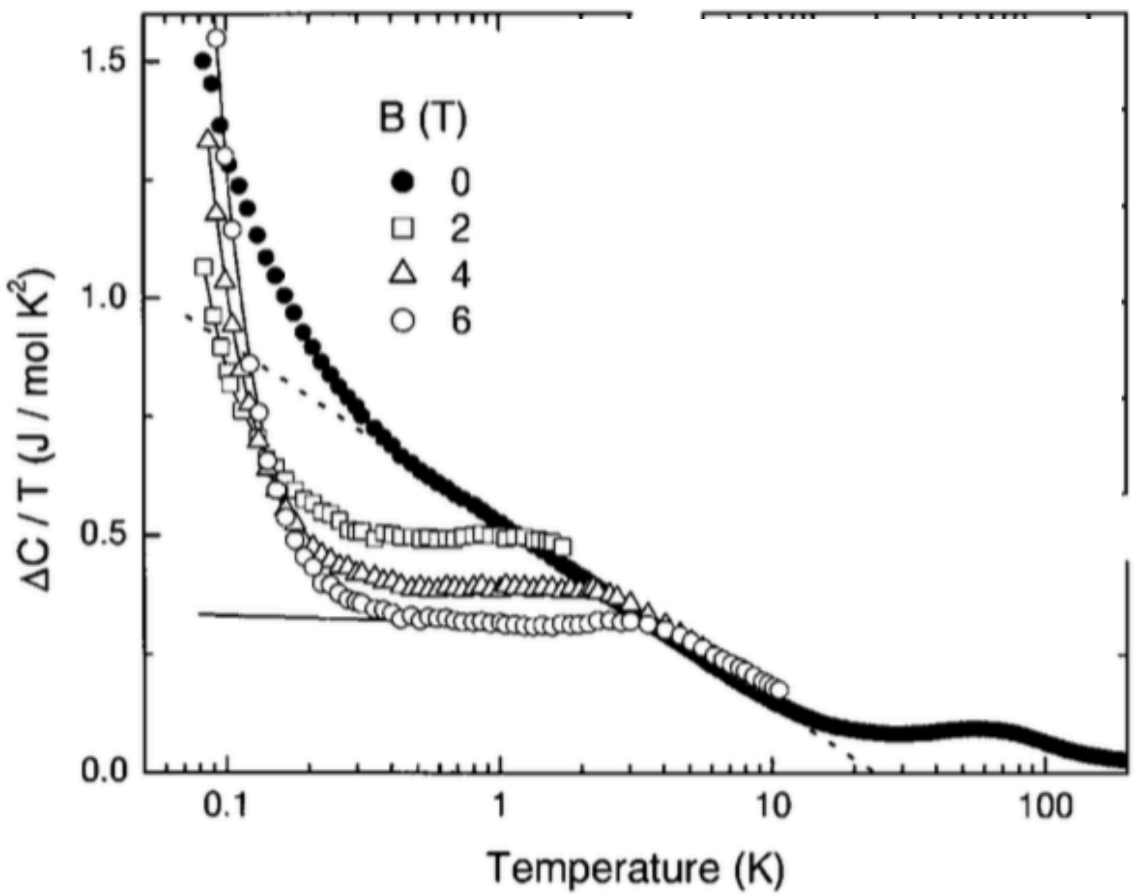


Particularly the iron-pnictide and heavy fermion examples, the region of T-linear resistivity resembles the quantum critical “fan” as one moves away from the critical point g_c.

Other properties

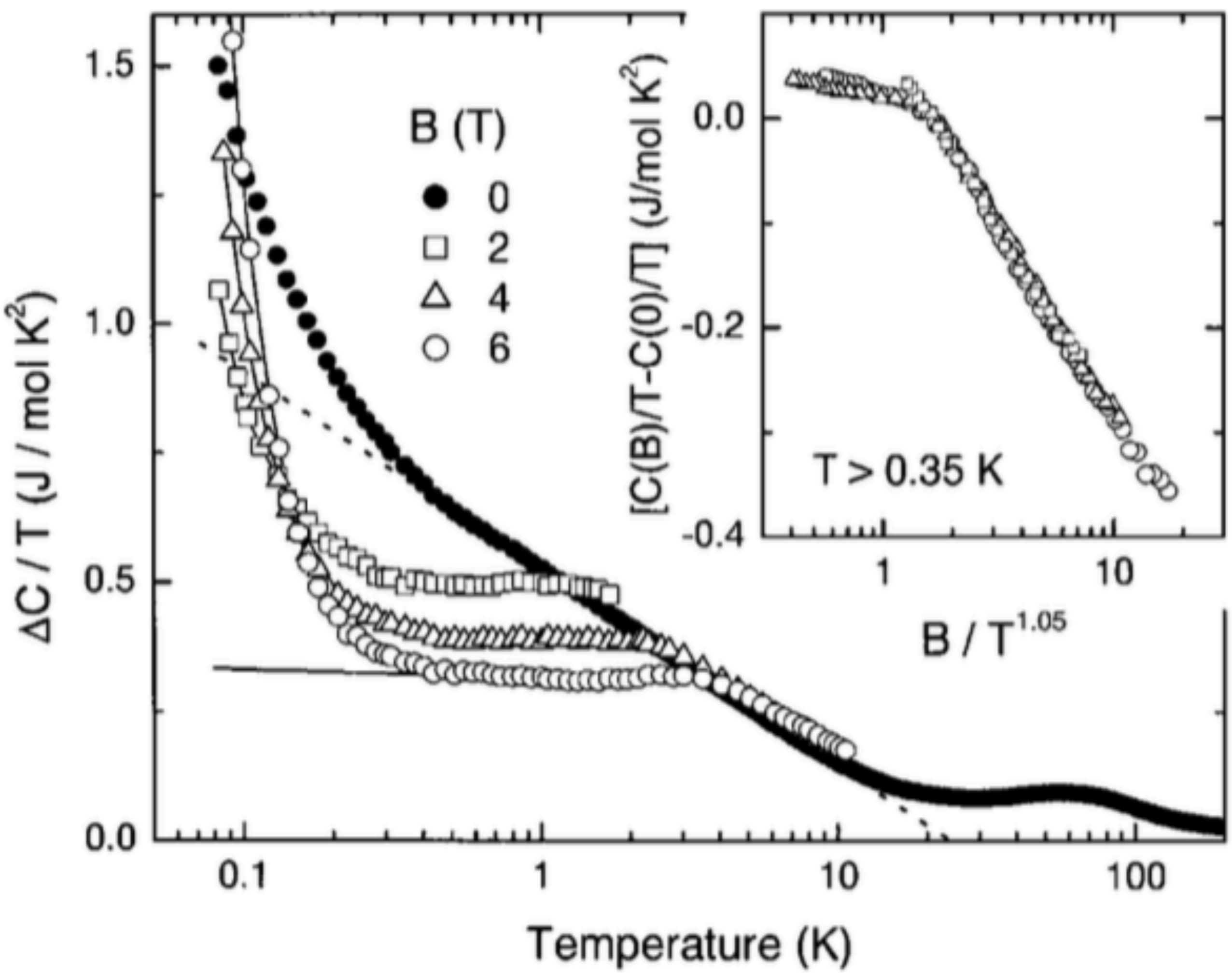
- On approach the quantum critical point we expect signatures of the critical exponents that define the critical point, particularly in the entropy carrying and magnetic degrees of freedom.
- Measurements of heat capacity, magnetic susceptibility, Nuclear Magnetic Resonance, and the effective mass are invaluable.

Heat capacity in $YbRh_2Si_2$



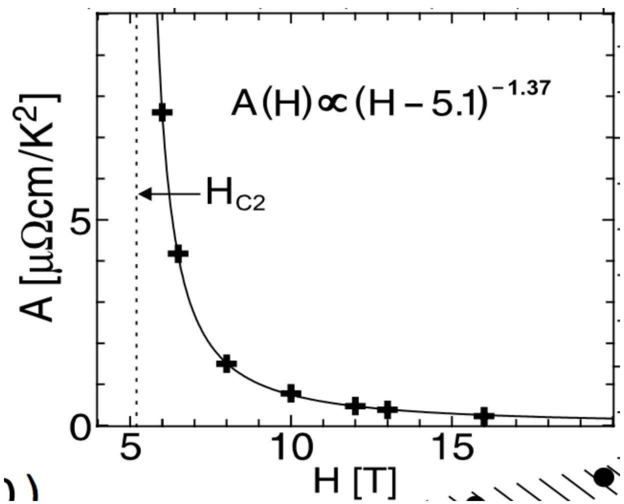
The logarithmic divergence of the heat capacity as T→0 is unexpected in a metal, but is consistent the approach to a quantum phase transition where the HC is characterized by critical exponents.

And there is associated scaling!



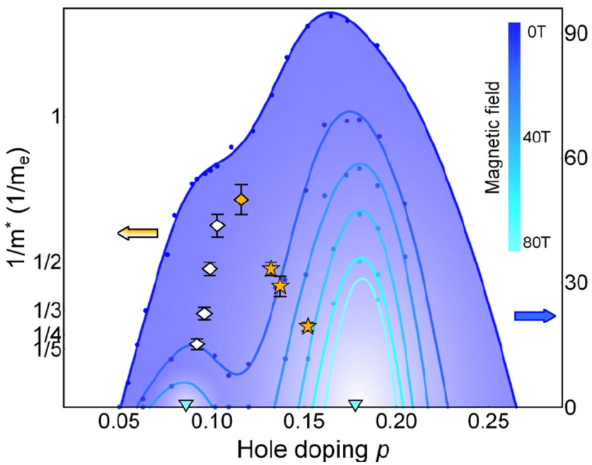
Divergence of the effective mass

Coefficient A of the FL resistivity
$\rho = \rho_0 + AT^2$



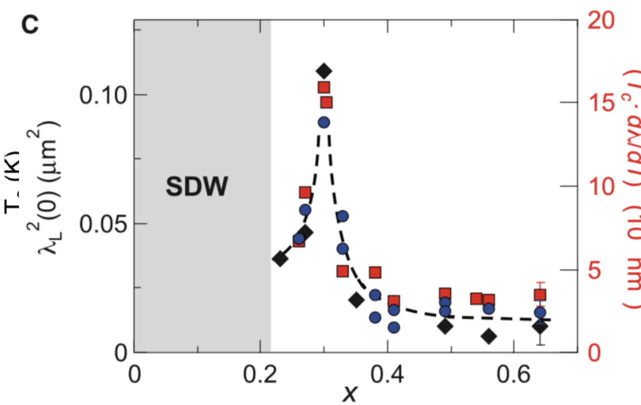
Paglione et al. on CeCoIn5
PRL 2003

Quantum oscillations



Ramshaw et al. on YBCO
Science 2015

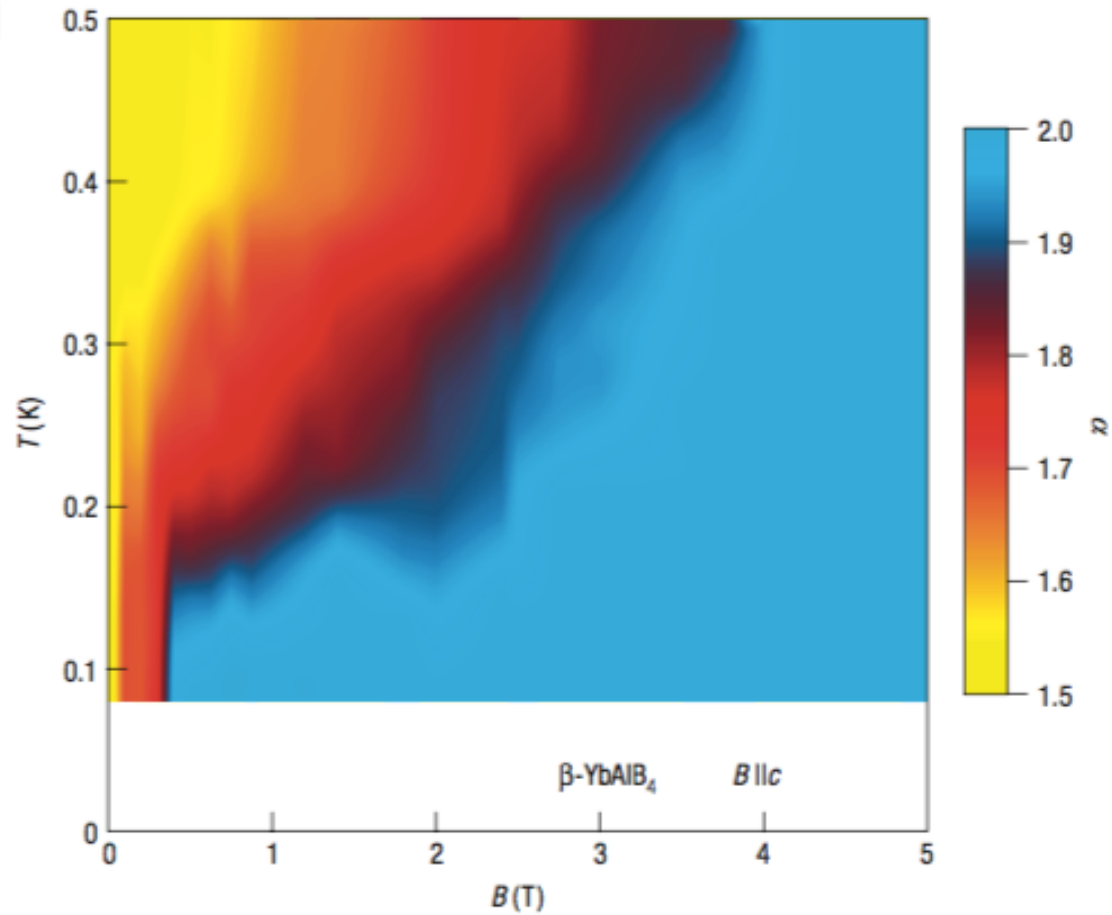
Penetration depth,
Sommerfeld coefficient….



Hashimoto et al. on BaFe2As2
Science 2012

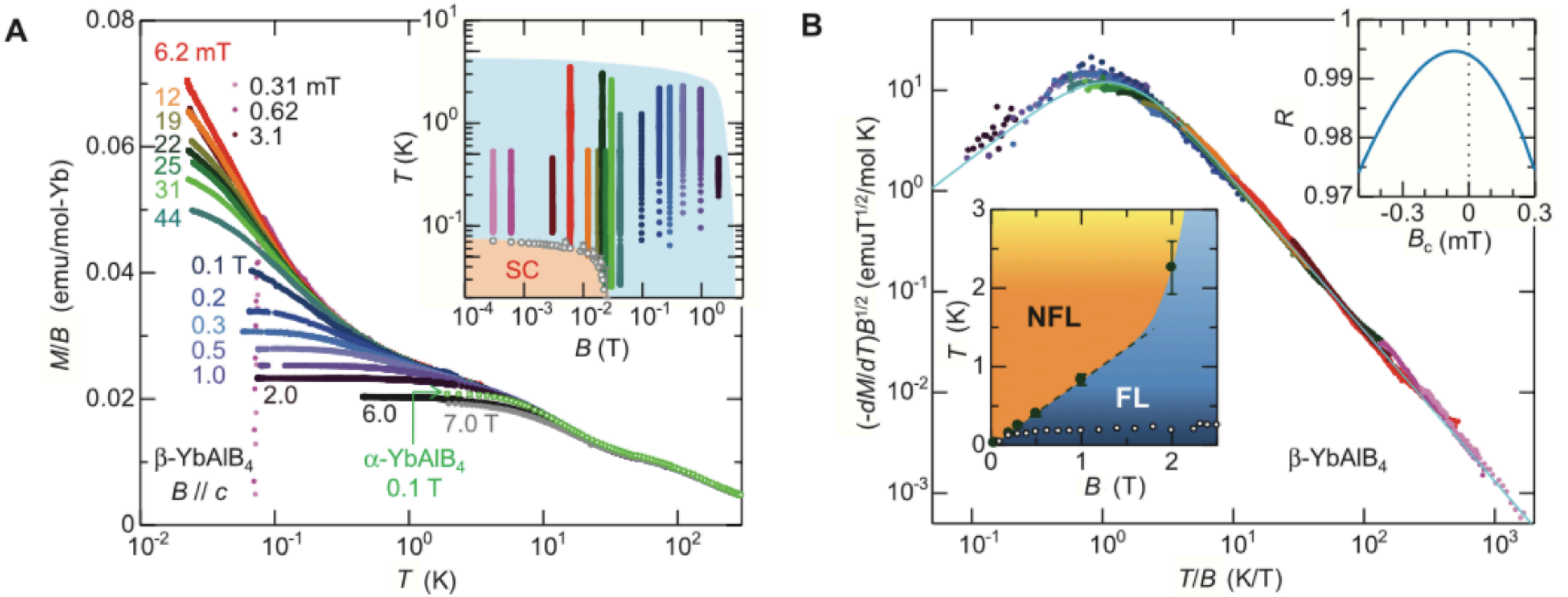
Recall that in Fermi liquid theory the effective mass renormalized by the strength of the electron-electron interactions. On approach to a QCP, the effective mass can become strongly enhanced as a result of these interactions.

Another neat example: $YbAlB_4$



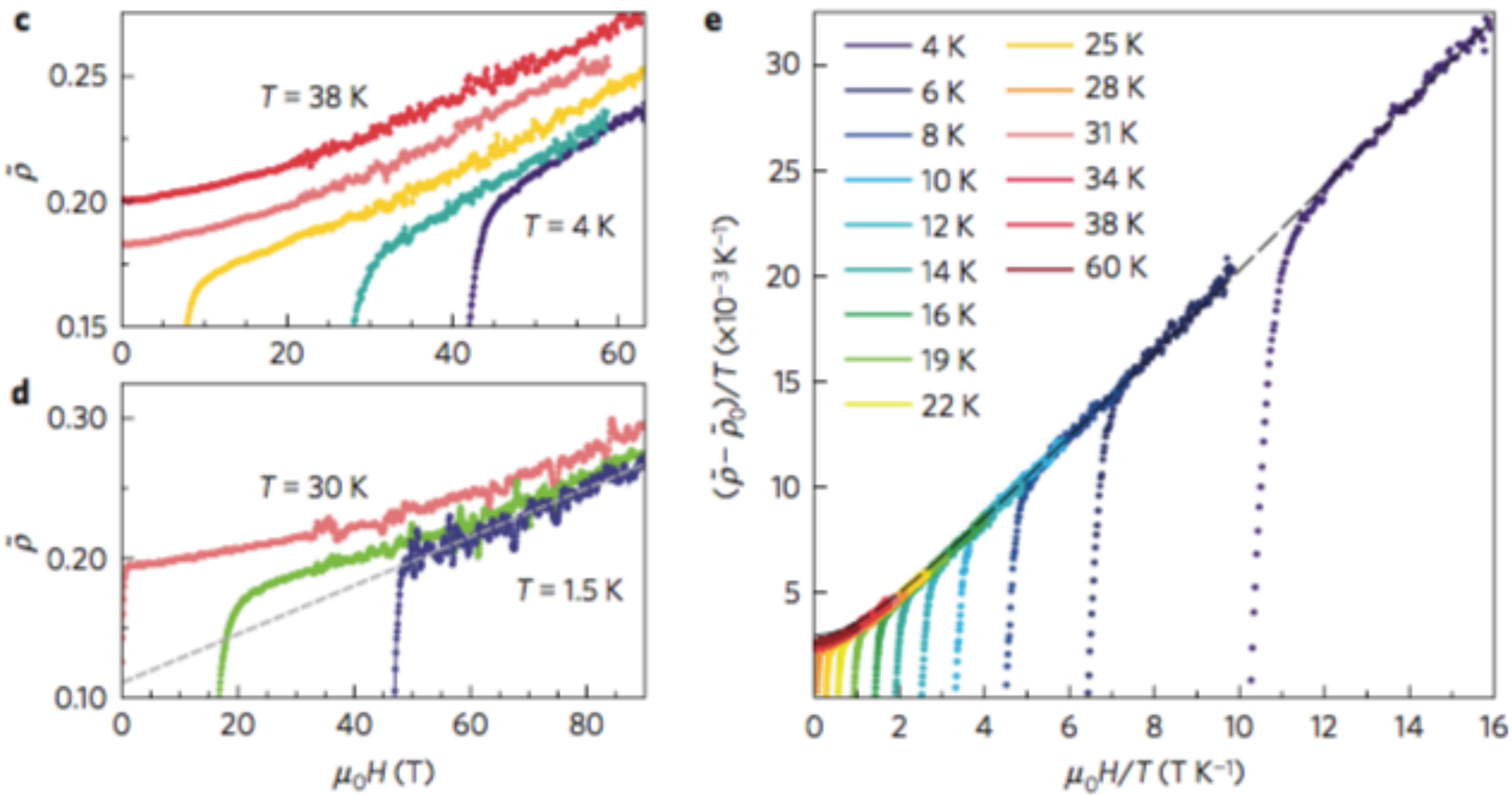
Non-Fermi liquid resisitvity but with a different power law than the “usual” one.
In YbAlB4 $\rho \sim T^{3/2}$

Another neat example: $YbAlB_4$



The magnetization, another thermodynamic quantity that should be governed by critical exponents near a QPT, shows B/T scaling behavior.

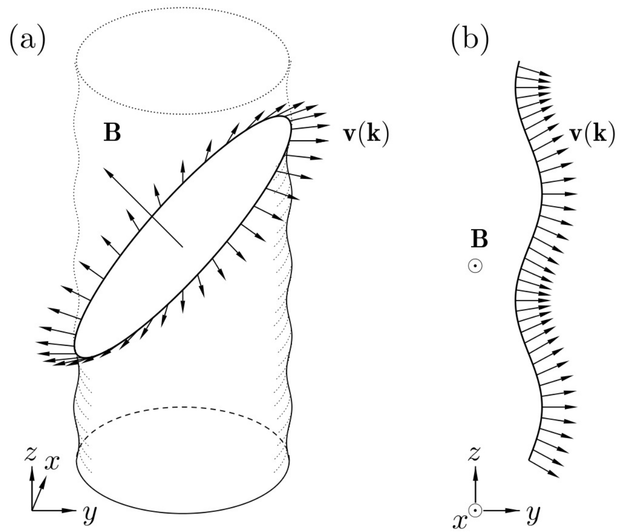
Examples of scaling behavior

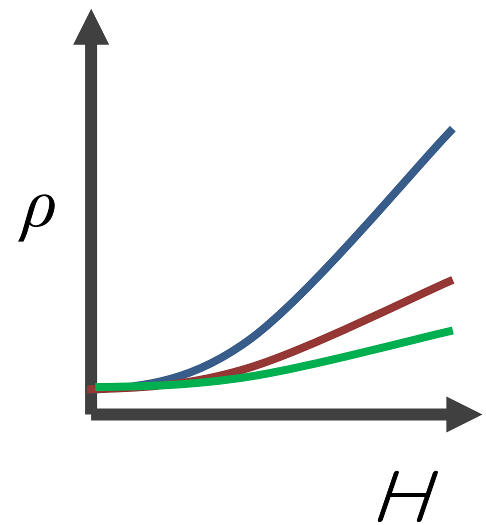


The magnetotransport of $BaFe_2(As_{1-x}P_x)_2$ shows a striking scaling in B/T, possibly indicating scale invariance.

Scaling is a tricky business….

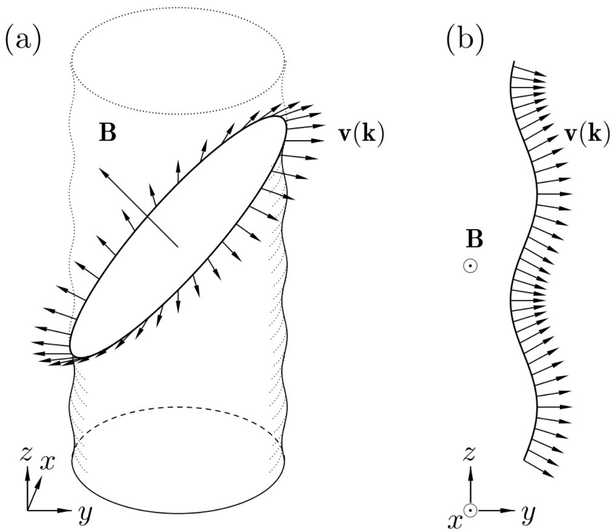
Kohler's Rule



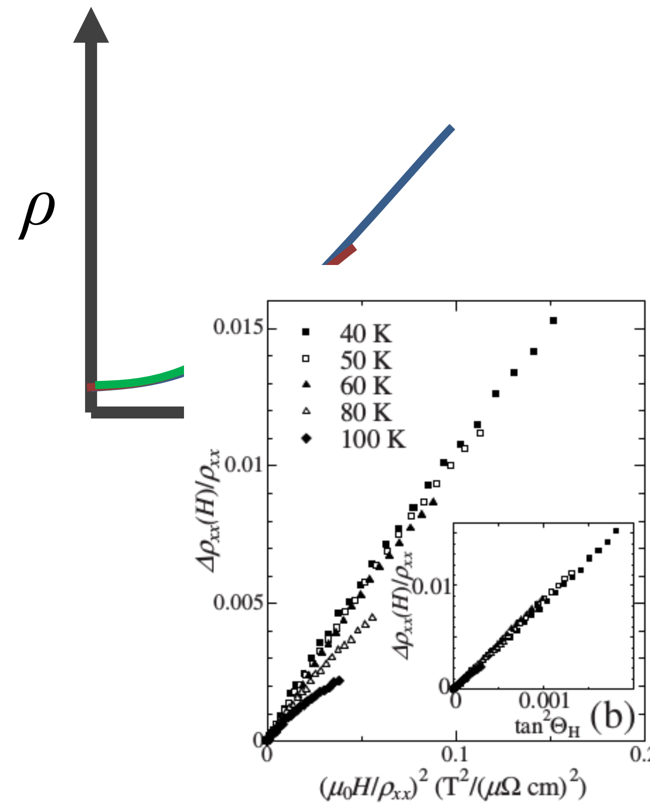


S. J. Blundell, Encyclopedia for Condensed Matter 2004

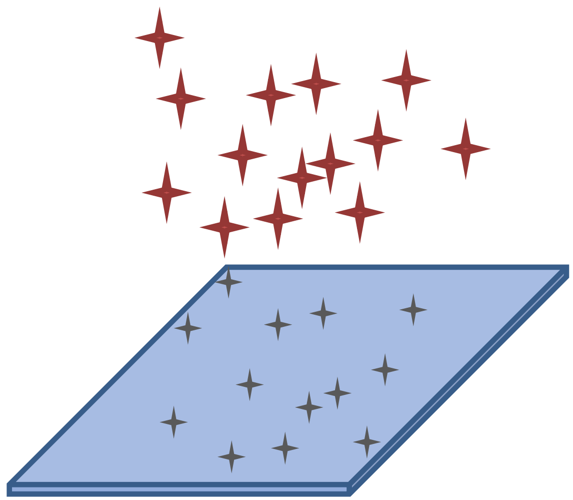
Kohler's Rule



S. J. Blundell, Encyclopedia for Condensed Matter 2004

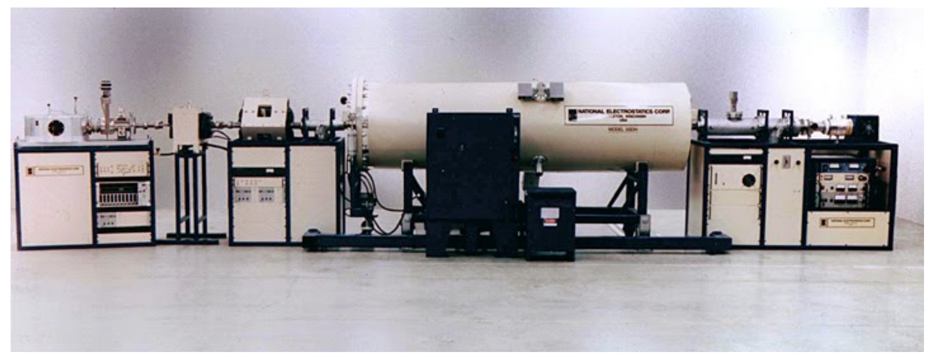


Systematic Disorder studies

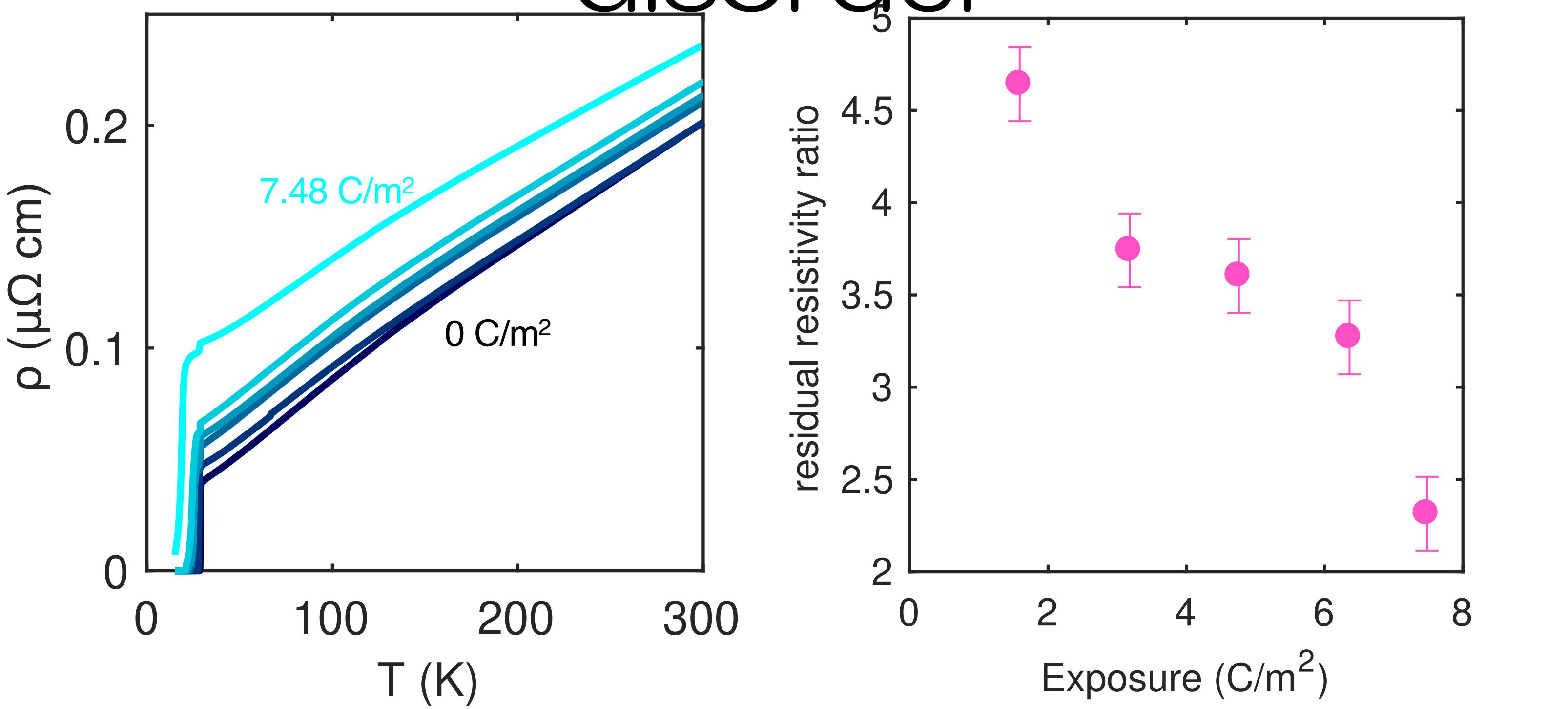


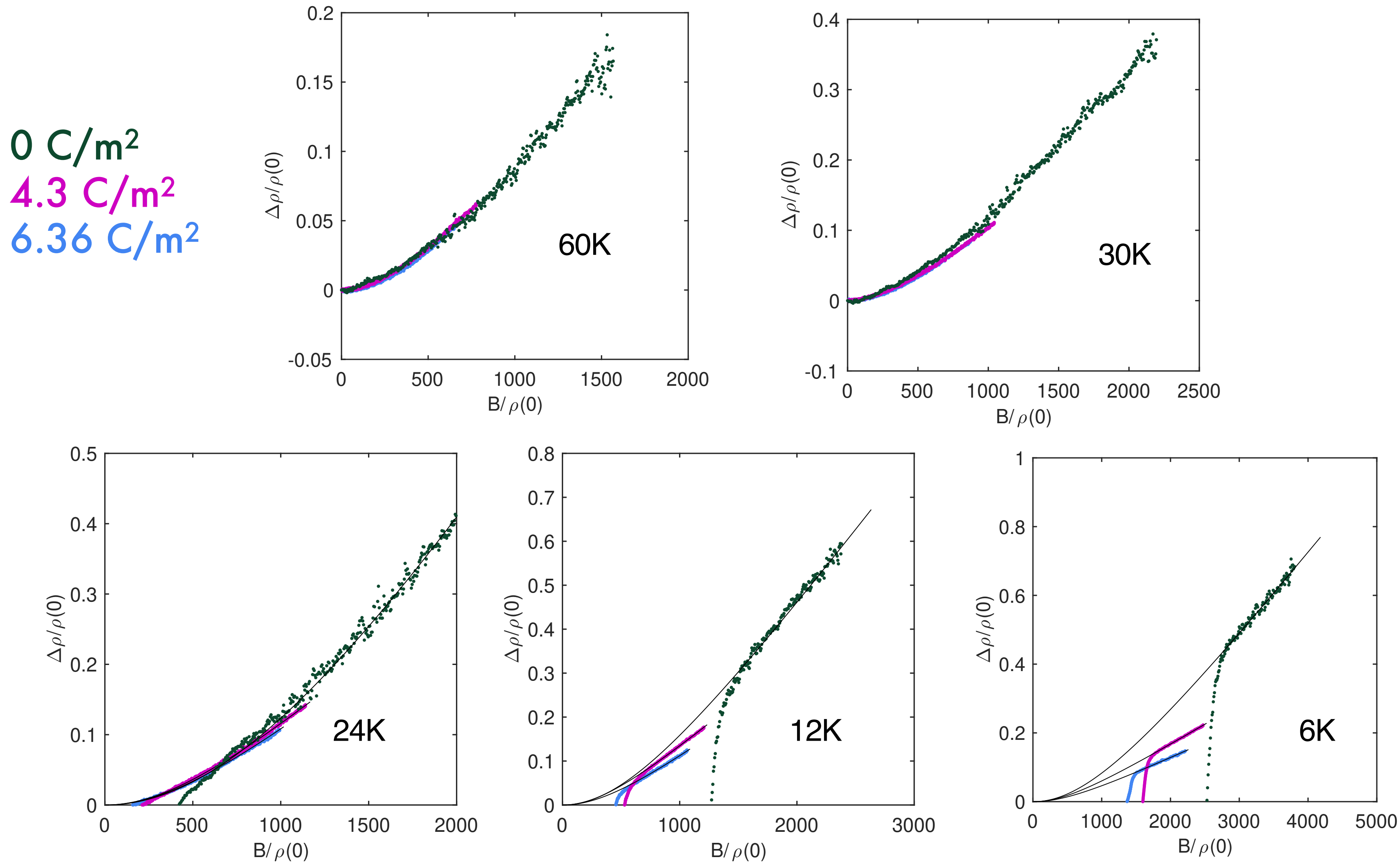
$<4\,\mu$m thick sample

Rutherford Backscattering Beamline (3 MeV) alpha particles, LBNL

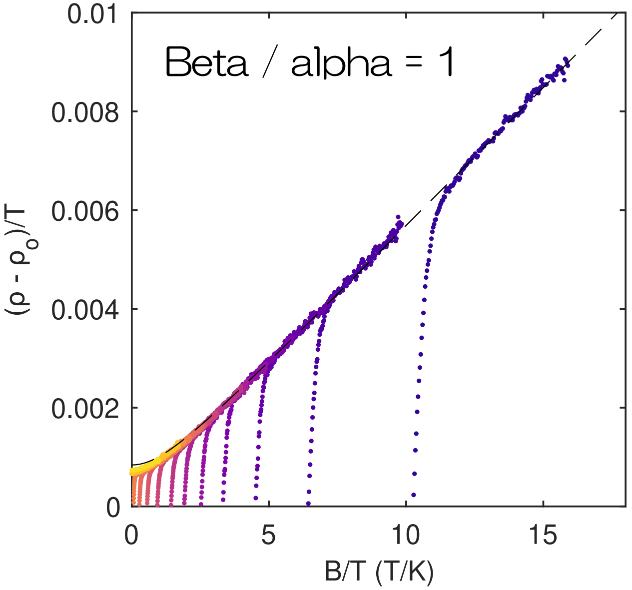


Resistivity as a function of disorder



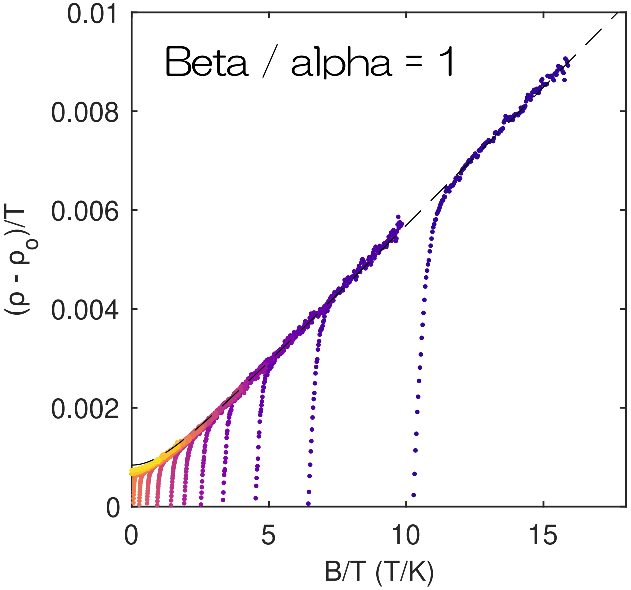

0 C/m²
4.3 C/m²
6.36 C/m²
60K
30K
24K
12K
6K
Δρ/ρ(0)
B/ρ(0)

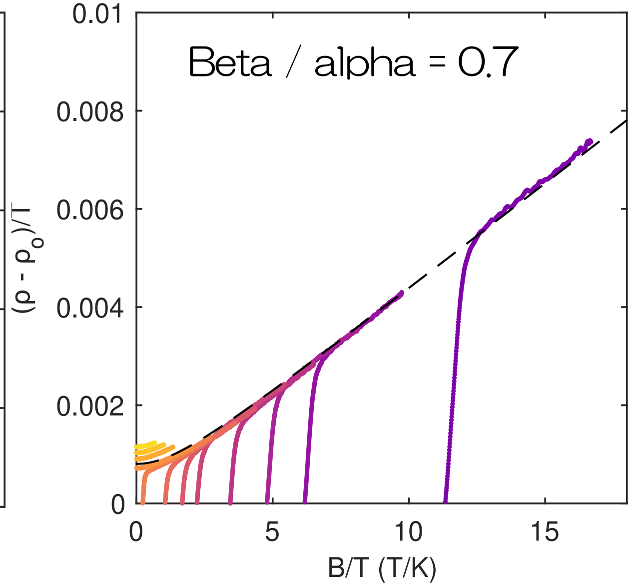
Disorder and B/T scaling



$0 C/m^2$ $4.3 C/m^2$ $6.36 C/m^2$

Disorder and B/T scaling



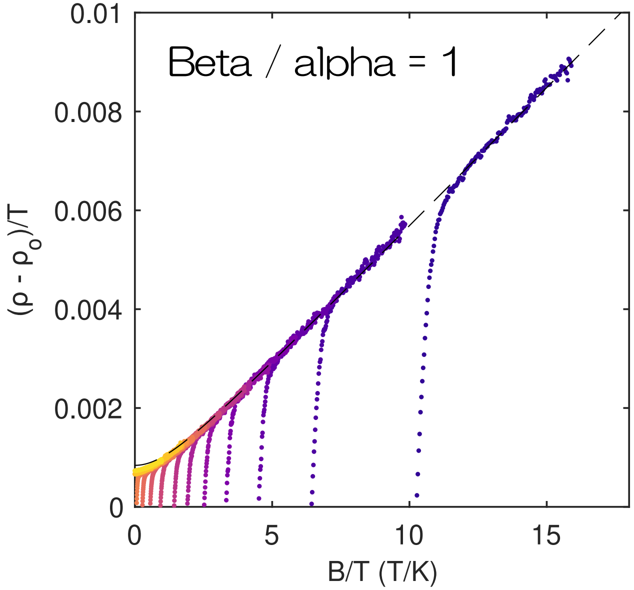


0C/m²

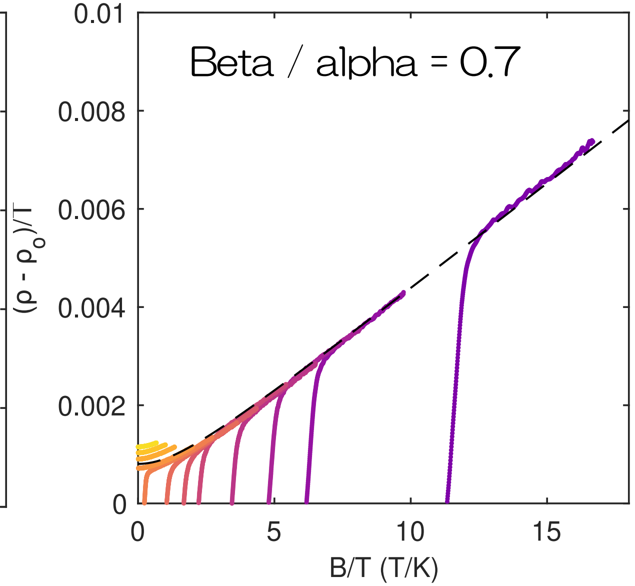
4.3C/m²

6.36C/m²

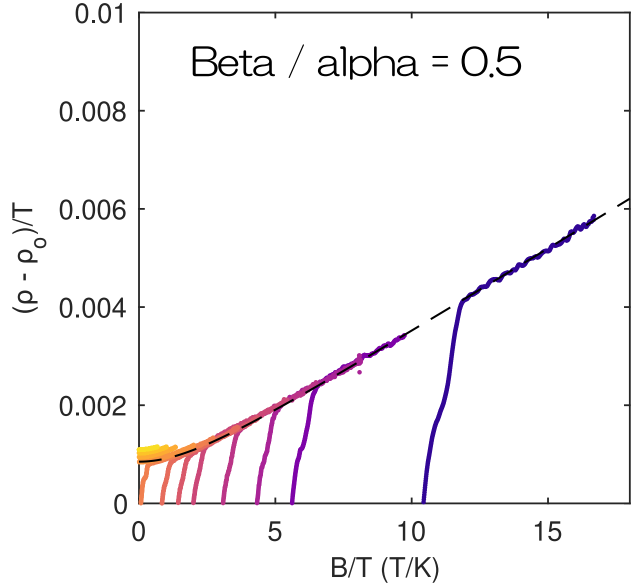
Disorder and B/T scaling



$0C/m^2$

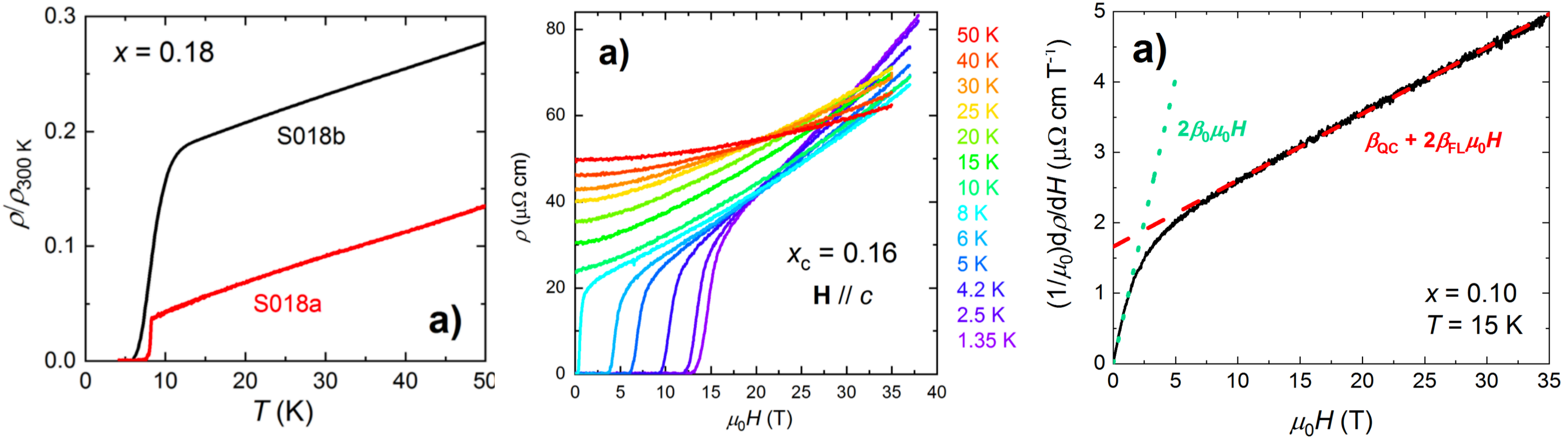


$4.3C/m^2$



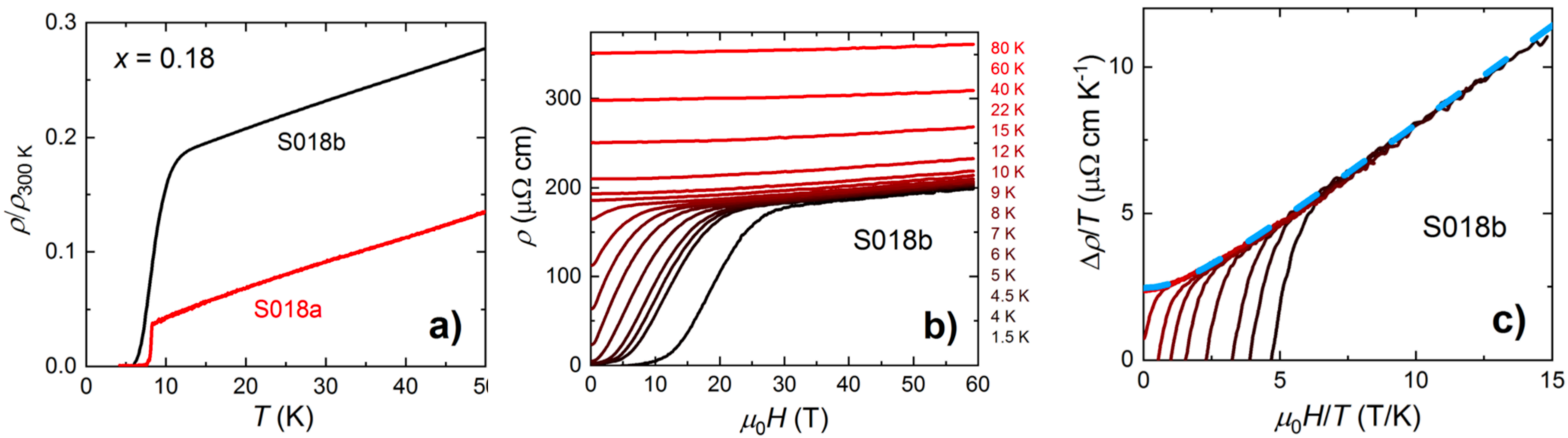
$6.36C/m^2$

Disorder and FeSe



S. Licciardello, N. E Hussey and JGA arxiv:1903.05679

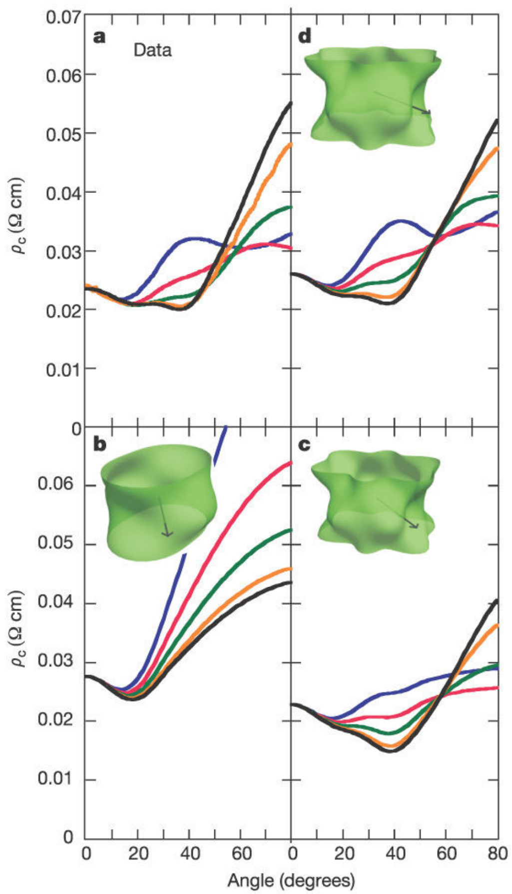
Disorder and FeSe



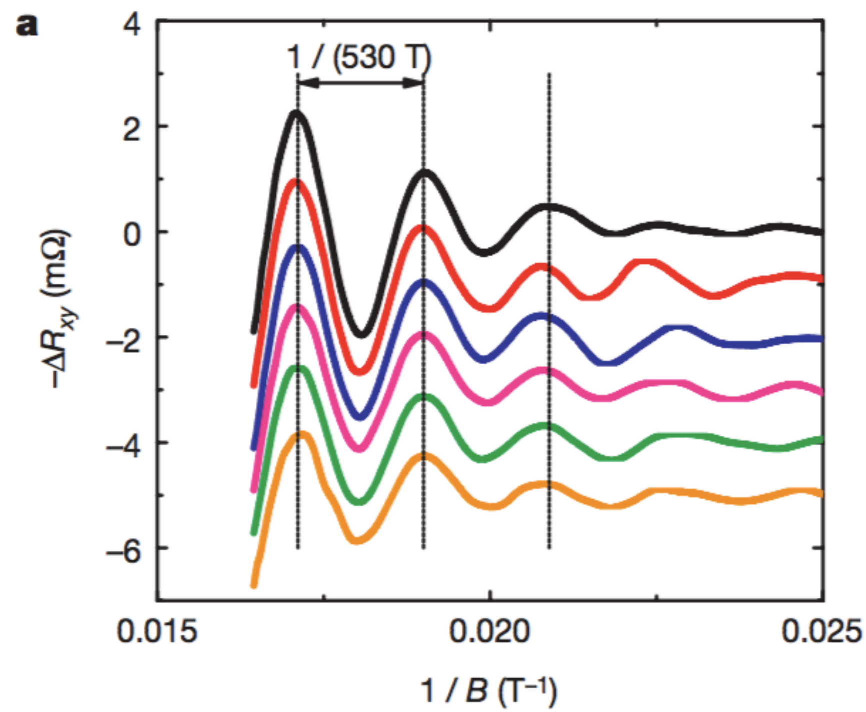
S. Licciardello, N. E Hussey and JGA arxiv:1903.05679

The elephant in the room

A big difficulty – there *is* another scale, E_F



Hussey et al. Nature 2003



Nicolas Doiron-Leyraud et al. Nature 2007